Rida Minahil GSA Maths Batch #85

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CSS 2025

Q 8. (d)

How many prime numbers are between each of the following pairs of numbers?

a. $\sqrt{3}$ and $\sqrt{120}$

$x = 3$

$y = 4$

4 is the nearest perfect ~~cube~~ square.

$$\sqrt{3} = \frac{x+y}{2\sqrt{y}} = \frac{3+4}{2\sqrt{4}}$$

$$\frac{7}{2(\sqrt{2^2})} = \frac{7}{4} = 1.75$$

$$\sqrt{120} = \frac{x+y}{2\sqrt{y}} = \frac{120+121}{2\sqrt{121}} = \frac{241}{2(\sqrt{11^2})}$$

$$= \frac{241}{22} = 10.95 \approx 11$$

Prime numbers between 1.75 and 11 are 2, 3, 5, 7.

b. $\sqrt[2]{10}$ and $\sqrt[2]{410}$

$x = 10$

$y = 9$

$$\sqrt[2]{10} = \frac{x+y}{2\sqrt{y}} = \frac{10+9}{2\sqrt{9}} = \frac{19}{2\sqrt{(3)^2}}$$

$$= \frac{19}{6} = 3.166$$

1.75
4 | 7
4
30
28
20
20
10.954
22 | 241
220
210
198
120
110
100
20
3.166
6 | 19
18
10
6
40
36
40
36
4

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$\sqrt[2]{410} = \frac{x+y}{2\sqrt{y}} = \frac{410+400}{2\sqrt{400}}$

$= \frac{810}{2(\sqrt{20^2})} = \frac{810}{40} = 20.25$

$Y = 400$

$40 \overline{)810}\ 20.25$

800

100

80

200

200

Prime numbers between 3.166 and 20.25 are

5, 7, 11, 13, 17, 19

c. $\sqrt[3]{10}$ and $\sqrt[3]{999}$

$x = 10$

$\therefore Y = 8$ nearest Perfect ~~square~~ cube

$\sqrt[3]{10} = \sqrt[3]{Y} + \frac{x-y}{3(\sqrt[3]{Y})^2} = \sqrt[3]{8} + \frac{10-8}{3(\sqrt[3]{8})^2}$

$\sqrt[3]{2^3} + \frac{2}{3(\sqrt[3]{2^3})^2} = 2 + \frac{2}{3(4)} = \frac{24}{12}$

$12 \overline{)4}\ 0.16$

12

80

72

80

72

$= 2 + \frac{2}{12} = 2 + 0.166 = 2.166$

$Y = 1000$

$x = 999$

$\sqrt[3]{999} = \sqrt[3]{Y} + \frac{x-y}{3(\sqrt[3]{Y})^2} = \sqrt[3]{1000} + \frac{999-1000}{3(\sqrt[3]{1000})^2}$

$= \sqrt[3]{10^3} + \frac{-1}{3(\sqrt[3]{10^3})^2} = 10 + \frac{(-1)}{3(100)} = 10 + \frac{-1}{300}$

$300 \overline{)1000}\ 0.0033$

900

1000

900

100

$= 10 - 0.0033 = 9.99$

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Prime numbers between 2.166 and 9.99 are 3, 5, 7.

d. $\sqrt[3]{28}$ and $\sqrt{120}$

$n = 28$, $y = 27$

$$\sqrt[3]{128} = \sqrt[3]{y} + \frac{n-y}{3(\sqrt[3]{y})^2} = \sqrt[3]{27} + \frac{28-27}{3(\sqrt[3]{27})^2}$$

$$= \sqrt[3]{3^3} + \frac{1}{3(\sqrt[3]{3^3})^2} = 3 + \frac{1}{3 \times 9} = 3 + \frac{1}{27}$$

$$= 3 + 0.037 = 3.037$$

$$\begin{array}{r} 0.037 \\ 27\overline{)100} \\ 81 \\ \hline 190 \\ 189 \\ \hline 100 \\ 81 \end{array}$$

$y = 121$, $x = 120$

$$\sqrt{120} = \frac{n+y}{2\sqrt{y}} = \frac{120+121}{2\sqrt{121}} = \frac{241}{2\sqrt{11^2}}$$

$$= \frac{241}{22} = 10.95$$

$$\begin{array}{r} 10.95 \\ 22\overline{)241} \\ 220 \\ \hline 210 \\ 198 \\ \hline 120 \\ 110 \end{array}$$

Prime numbers between 3.037 and 10.95 are 5, 7.

e. $\sqrt[2]{8}$ and $\sqrt{400}$

$n = 8$, $y = 9$

$$\sqrt[2]{8} = \frac{n+y}{2\sqrt{y}} = \frac{8+9}{2\sqrt{9}} = \frac{17}{2(\sqrt{3^2})} = \frac{17}{6}$$

$$= 2.83$$

$$\begin{array}{r} 2.83 \\ 6\overline{)17} \\ 12 \\ \hline 50 \\ 48 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

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$\sqrt{400} = \sqrt{20^2} = 20$

Prime numbers between 2.83 and 20 are
3, 5, 7, 11, 13, 17, 19.