CSS 2020

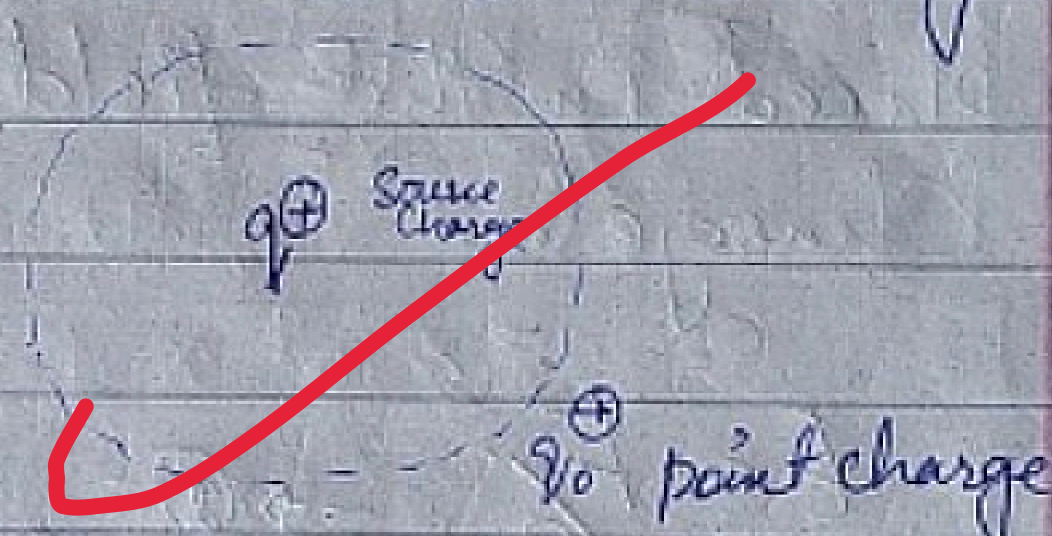
Part II

Q2 (a) Discuss the electric field of point charges, keeping in view the magnitude of force acting on test charge according to Coulomb's law.

Ans Electric field of a point charge:

(a) <u>Definition of Electric field</u>:-

It is a space or a region around a charge.



(b) Coulomb's law:

<u>Statement</u>:

The force of attraction or repulsion between two point charges varies directly with the magnitude of charges and inversely as square of the distance between two charges.

$$F = \frac{k\,|q|\,|q_0|}{r^2} \rightarrow (1)$$

(c) Electric field of a point Charge:

The electric field "E" created by a point charge "q" is defined as,

" The force experienced by a small positive test charge (q_0) placed in the field, per unit Charge".

$$E = F/q_0 \rightarrow (2)$$

Put eq (1) in eq (2) we have

$$E = \frac{\frac{k\,|q|\,|q_0|}{r^2}}{q_0}$$

$$E = \frac{k\,|q|}{r^2} \rightarrow (3)$$

This equation shows that the electric field is directly proportional to the magnitude of the source charge (q) and inversely proportional to the square of the distance from the charge.

(d) Magnitude of Force on a Test Charge

Attempt you giving statements

As $E = F/q_0$

$$F = Eq_0 \longrightarrow (4)$$

Now put eq (3) in eq (4), we have,

$$F = q_0\left(\frac{k|q|}{r^2}\right) = \boxed{k\frac{|q||q_0|}{r^2}}$$

This shows that the force acting on the test charge is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

(b) Find out the electric field due to charge of 2e at a distance of 26.5×10^{-3} m. ($\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$ and $e = 1.60 \times 10^{-12}\ C$).

Solution

To find the electric field due to a ~~po~~ charge, we use the formula,

$$E = \frac{k \cdot |q|}{r^2} \rightarrow ①$$

Put the values in eq ①

$$E = \frac{8.988 \times 10^{9} \times 1.60 \times 10^{-12}}{(26.5 \times 10^{-13})^2}$$

10^{-12+9}

$$E = \frac{14.381 \times 10^{-3}}{702.25 \times 10^{-26}}$$

10^{-3+26}

$$\cancel{E = 2.048 \times 10^{22}}$$

$$E = 0.02048 \times 10^{-3+26}$$

$$E = 2.048 \times 10^{-2} \times 10^{23}$$

$$\boxed{E = 2.048 \times 10^{21}\ N/C}$$

$10^{23-2} = 21$

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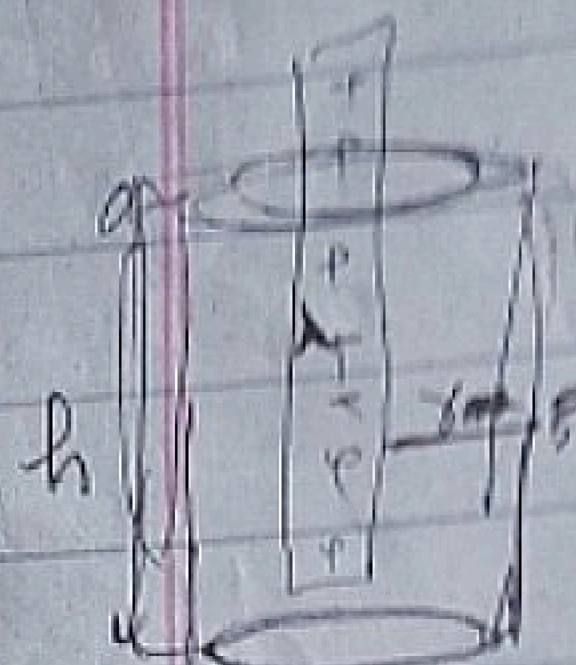
Q2(a) Consider an infinitely long cylindrical insulating shell of inner radius a, and outer radius b, and has a uniform volume charge ~~charge~~ density ρ. If a line of charge density λ is placed along the axis of the shell, then determine the electric field intensity at a point r such that (ii) $a < r < b$ and (iii) $r > b$
(i) $r < a$



Ans By definition of Gauss's law

$$\phi_e = \frac{1}{\varepsilon_0}(Q) \rightarrow ①$$

By definition of electric flux

$$\phi_e = \vec{E} \cdot \vec{A} \text{ or } \phi_e = \int E \cdot da \rightarrow ②$$

Comparing ① & ②

$$\int E \cdot da = \frac{1}{\varepsilon_0}(Q)$$

As charge is uniformly distributed so, E = constant

$$\boxed{E \int da = \frac{1}{\varepsilon_0}(Q)} \rightarrow ③$$

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Discuss the Lorentz force

②

As linear charge density $= \lambda = Q/L$

$\Rightarrow Q = \lambda L \Rightarrow \boxed{Q = \lambda h}$ put in eq ③

③ $\Rightarrow E \int da = \frac{1}{\varepsilon_0} (\lambda h)$

$\because$ Area of cylinder $= 2\pi r . h$

$$E . (2\pi r . h) = \frac{1}{\varepsilon_0} (\lambda h)$$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$ Electric field due to a line of charge (infinite).

So we have the electric field of the line,

$$E_1(r) = \frac{\lambda}{2\pi \varepsilon_0 r} , r > 0$$

④ $r < a$

In this case,

$$E_2 . 2\pi r h = \frac{0 . \pi r^2 h}{\varepsilon_0} = 0$$

Hence, $E_2(r) = 0$, $r < a$

(ii) $a < r < b$

In this case,

$$E_2 \cdot 2\pi r h = \rho \cdot \frac{\pi (r^2 - a^2) h}{\varepsilon_0}$$

$$E_2(r) = \rho \cdot \frac{(r^2 - a^2)}{2\varepsilon_0 r}, \quad a < r < b$$

The total electric field is:

$$E(r) = E_1(r) + E_2(r) = \frac{\rho \cdot (r^2 - a^2) + \lambda}{2\varepsilon_0 r}, \quad a < r < b$$

(iii) $r > b$

In this case,

$$E_2 \cdot 2\pi r h = \rho \cdot \frac{\pi (b^2 - a^2) h}{\varepsilon_0}$$

Hence,

$$E_2(r) = \rho \cdot \frac{(b^2 - a^2)}{2\varepsilon_0 r}, \quad r > b$$

The total electric field is:

$$E(r) = E_1(r) + E_2(r) = \frac{\rho \cdot (b^2 - a^2) + \lambda}{2\varepsilon_0 r}, \quad r > b$$

(b) Discuss the Lorentz force.

Ans Introduction:-

The first derivation of the Lorentz force is commonly attributed to Oliver Heaviside in 1889. Hendrik Lorentz derived it a few years after Heaviside.

Definition:

Force on moving charged partical placed in electromagnetic field is known as Lorentz force.

Explaination:-

The force exerted on a charged particle q moving with velocity v through an electric E and magnetic field B. The entire electromagnetic force F on the charged particle is called the

Lorentz Force and is given by

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B}) + q\vec{E}}$$

- $\vec{v}$ = velocity of Charge q
- $q(\vec{v} \times \vec{B})$ = Force on q by magnetic field
- $q\vec{E}$ = Force on q by electric field.

$$\boxed{\vec{F} = q[\vec{v} \times \vec{B} + \vec{E}]}$$

Good answers!!

Lorentz Force Properties

- $F = 0$ unless charge is moving
- $F = 0$ if velocity is parallel to field.
- F = maximum if velocity is $\perp$ to field.
- $F \neq 0$ only if charge crosses B field lines
- If q, $\vec{v}$, or $\vec{B}$ reverse, direction of $\vec{F}$ reverses.

$\longleftrightarrow$