CSS - 2019 (Q) Part II

Q2 (a) Derive an expression for the torque and potential energy of an electric dipole in an electric field.

Solution (a) Definition of Torque:-

Torque is a measure of the force that can cause an object to rotate about an axis. It is a vector quantity.

Torque vector's magnitude:-

$$\tau = F \times r$$

$$\tau = F \times r \times \sin\theta$$

where, r = Position vector (a vector from the point about which the torque is being measured to the point where the force is applied).

F = Force vector, θ = angle b/w the force vector and the lever arm vector.

(b) Electric dipole:- Pair of opposite electric charges with equal magnitudes that are lined apart by "d".

Electric Dipole moment:-
The product of the magnitude of these charges (q) and their seperation from each other (d).
Direction : From the negative charge to the positive charge. Unit = cm
Formula: $p = q \times d$
→ It is a vector quantity
where "d" is a vector ~~quantity~~ pointing from the negative charge to the positive charge.

① Torque on a Electric Dipole in an Electric field:
When an electric dipole is placed in a uniform electric field E, it experiences a torque τ. The torque tends to align the dipole with the electric field. The expression for torque is given by the

Derivation??

cross product of the dipole moment p and the electric field E:

$$\tau = p \times E$$

(dipole moment)

We have magnitude of the torque is:

$$|\tau| = pE \sin\theta$$

where:

- $p = |P|$ is the magnitude of the dipole moment.
- $E = |E|$ is the magnitude of the electric field
- θ is the angle between p and E

② Potential Energy of an Electric Dipole of an Electric Field:

The potential energy U of an electric dipole in an external electric field depends on its orientation with respect to the field.

The potential energy is given by:

$$U = -P \cdot E$$

Magnitude of Potential Energy:-

The dot product can be expanded as:

$$\boxed{U = -PE\cos\theta}$$

~~where~~

Ans

Part (b) Show that the energy density of a parallel plate capacitor with dielectric medium b/w them is directly proportional to the square of electric field intensity.

Solution:

To show that the energy density of a parallel plate capacitor with a dielectric medium between the plates is directly proportional to the square of the electric field intensity

Firstly,

(1) Energy stored in a capacitor, we have

$$C = k\varepsilon_0 \frac{A}{d} \rightarrow (1)$$

And the energy U stored in a capacitor when it is charged to a potential difference V is given by:

$$U = \frac{1}{2} C V^2 \rightarrow (2)$$

Now substitute the expression for capacitance in eq (2)

$$U = \frac{1}{2}\left(k\varepsilon_0 \frac{A}{d}\right)V^2 \rightarrow (3)$$

(2) Now, we relate potential difference to electric field, we have,

$$E = \frac{V}{d}$$

or $V = Ed$ — (4)

Put eq (4) in eq (3) we have

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$$U = \frac{1}{2} k\varepsilon_0 A d E^2 \rightarrow (5)$$

(3) Now Expression for energy density:

The energy density u is the energy stored per unit volume b/w the capacitor plates.

The volume V b/w the plates is,

$$V_{volume} = A \cdot d$$

So the energy density u is

$$u = \frac{U}{V_{volume}} = \frac{\frac{1}{2} k\varepsilon_0 A d E^2}{A d}$$

$$u = \frac{1}{2} k\varepsilon_0 E^2$$

Ans

(c) In a microwave oven torque acting on an electric dipole is responsible for the production of heat. Comment.

Ans How Torque Acts on Electric Dipoles in a Microwave Oven:

In a microwave oven, heat is produced by the interaction of microwave radiation with polar molecules, like water, in the food. These molecules have an electric dipole moment due to their uneven charge distribution. A microwave create an oscillating electric field that exerts a torque on the dipoles, causing them to rotate rapidly as they try to align with the changing field.

This rapid rotation leads to collisions and friction among the molecules, which converts the kinetic energy of their motion into thermal energy (heat). Thus, the torque acting on electric dipoles due to the microwave's electric field is directly responsible for heating the food.