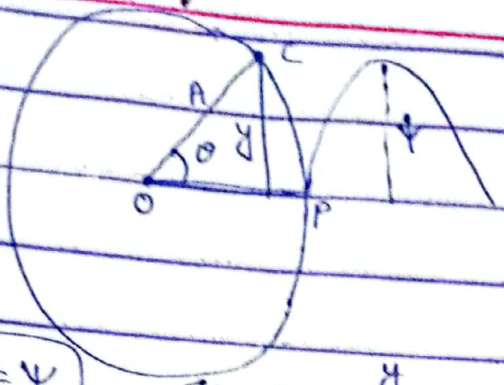
Schrodinger wave Equation.



Hints

$y = \psi$
$\theta = \omega t$
$\omega = 2\pi\nu$
$\nu = \frac{v}{\lambda}$
$t = \frac{x}{v}$

$\boxed{y = \psi}$ $\sin\theta = \frac{y}{A} = \frac{\psi}{A} = \psi = A\sin\theta$

$\psi = A\sin\theta$

$\psi = A\sin\omega t = A\sin 2\pi\nu t = A\sin 2\pi\frac{v}{\lambda}\cdot\frac{x}{v} = \boxed{A\sin\frac{2\pi x}{\lambda}}$

$$\boxed{\psi = A\sin\frac{2\pi x}{\lambda}} \quad \text{— (1)}$$

Differentiate eq 1

in differentiation $\boxed{\sin \rightarrow \cos}$

$$\frac{d\psi}{dx} = \frac{2\pi A}{\lambda}\left[\cos\frac{2\pi x}{\lambda}\right] \quad \text{— (2)}$$

~~Double~~ Differentiate eq 2

$$\frac{d\psi^2}{dx^2} = \frac{2\pi A}{\lambda}\times\left[\frac{2\pi}{\lambda}\right]\left[-\sin\frac{2\pi x}{\lambda}\right]$$

$\boxed{\cos \rightarrow -\sin}$

$$\frac{d\psi^2}{dx^2} = -\frac{4\pi^2}{\lambda^2}\left[A\sin\frac{2\pi x}{\lambda}\right]$$

$$\frac{d\psi^2}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi$$

$$\boxed{\frac{d\psi^2}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0} \quad \text{— (3)}$$

Eq (3) is a standing wave eq. convert it into an equation that can be applied electron by substituting λ

$\lambda = \frac{h}{mv} \Rightarrow \lambda^2 = \frac{h^2}{m^2 v^2}$ — (4)

For substitution of v^2

$K.E = \frac{1}{2} m v^2$ $\quad$ $\boxed{K.E = \mathcal{E}}$

$\boxed{\frac{2\mathcal{E}}{m} = v^2}$

putting value of v^2 into eq (4)

$\lambda^2 = \frac{h^2}{m^2 \frac{2\mathcal{E}}{m}} = \frac{h^2}{m 2\mathcal{E}}$

putting value of λ^2 in eq (3)

$\frac{d\psi^2}{dx^2} + \frac{4\pi^2}{\frac{h^2}{m 2\mathcal{E}}} \psi = 0$ 10

$\frac{d\psi^2}{dx^2} + \frac{8\pi^2 m}{h^2} \mathcal{E} \psi = 0$

$\boxed{\begin{aligned} E &= K.E + P.E \\ E &= \mathcal{E} + P \\ \mathcal{E} &= E - P \end{aligned}}$

$\boxed{\frac{d\psi^2}{dx^2} + \frac{8\pi^2 m}{h^2} (E - P)\psi = 0}$ → (6)

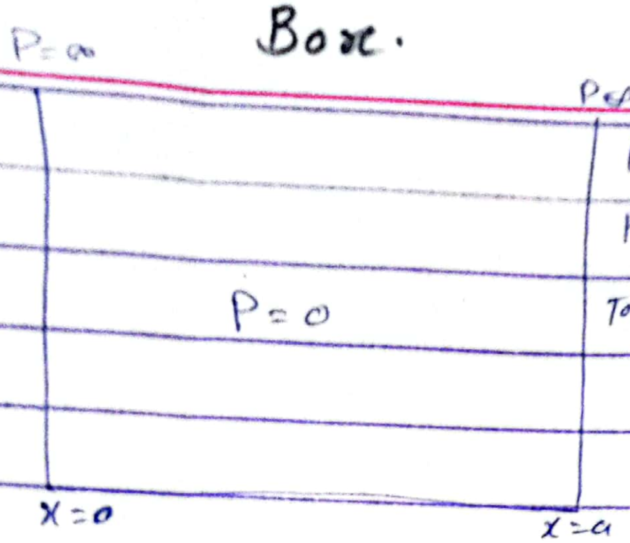
Eq 6 (Schrongi) Schrodinger Wave Equation representing particle moving along one dimension in space.

$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m}{h^2} (E - P)\psi = 0$ — (7)

Eq (7) is a Schrodinger wave equation for a particle in three dimensions.

$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - P)\psi = 0$ $\quad$ ∇^2 = [redacted] operator.
Laplacian

Motion of Particle in One Dimensional Box.



P.E = 0 ⟹ inside box.

P.E = ∞ At walls and outside of box.

Total Energy (E) = Constant.

A particle is moving along x-axis in above box. So, Schrodinger wave equation for one Dimension will be applicable.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-P)\psi = 0$$

As $P = 0$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

$\frac{8\pi^2 m}{h^2}E = k^2$ is a constant.

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{(1)}$$

→ It is a second order differential eq. It has following solution.

$$\psi = A\sin kx + B\cos kx. \quad -(2)$$

Value of k^2 for eq (1) can be calculated from eq (2) by applying Boundary Conditions.

i) When $x = 0$ $\psi = 0$

$\psi = A\sin kx + B\cos kx$

$0 = A\sin k(0) + B\cos k(0)$

$\sin 0 \to 0$
$\cos 0 \to 1$

$0 = 0 + B \Rightarrow B = 0$

Putting value of B in eq ②

$\psi = A \sin kx + 0$

$\psi = A \sin kx$

ii) When $x = a$ $\psi = 0$

$\psi = A \sin kx + B \cos k(x)$

$0 = A \sin ka + B \cos ka \leftarrow \boxed{B = 0}$

$\boxed{A \sin ka = 0} \Rightarrow A \neq 0$

$\sin ka = 0$ $\qquad \cancel{\sin} \, n\pi = \cancel{\sin} \, ka$

$\sin n\pi = 0$ $\qquad n\pi = ka$

$$\boxed{k = \frac{n\pi}{a}} \quad \text{—③}$$

$$k^2 = \frac{n^2\pi^2}{a^2}$$

By camparing values of k^2

$$\frac{n^2 \cancel{\pi^2}}{a^2} = \frac{8\cancel{\pi^2} m}{h^2} E$$

$$\frac{n^2}{a^2} = \frac{8m}{h^2} E$$

$$\boxed{E = \frac{n^2 h^2}{8ma^2}}$$

$$k^2 = \frac{n^2\pi^2}{a^2}$$

$$\boxed{k^2 = \frac{8\pi^2 m}{h^2} E}$$

Value of k^2 taken at start.

This equation is used to calculate energy of electron moving along x axis in one dimensional box.