Eigenfunctions

Eigenfunctions are a fundamental concept in math and physics that provide a powerful tool in solving a wide range of problems involving linear operators.

In Q.M, eigenfunctions are used to describe the wave-like behavior of particles and the probability of finding a particle in a particular state.

For Example

The wave function of an electron in an atom is an eigenfunction of the Hamiltonian operator, which describes the total energy of the electron.

Explanation

The wave function for a given physical system contains the measurable information about the system. To obtain specific values for physical parameters, for example energy, you operate on

the wavefunction with the Q Mechanical operator associated with that parameter. The operator associated with energy is the Hamiltonian, and the operation on the wavefunction is the Schrodinger equation.

Solutions exist for the time-independent Schrodinger equation only for certain values of energy, and these values are called eigenvalues of energy.

"Corresponding to each eigenvalue is an eigenfunction". The solution to Schrodinger equation for a given energy E_i envolves also finding the specific function ψ_i which describes that energy state. The solution of the time independant schrodinger equation takes the form

$$H_{op}\,\psi_i = E_i\,\psi_i$$

The eigenvalue concept is not limited to energy. when applied to a general operator Q, it can take the form

$$Q_{op}\,\psi_i = q_i\,\psi_i$$

(Q_{op}: operator; q_i: eigenvalue; ψ_i: eigenfunction)

If the function ψ_i is an eigenfunction for the operator. The eigenvalue q_i may be discrete, and in such cases we can say that the physical variable is "quantized" and that the index i plays the role of a "quantum num" which characterizes that state.

Eigenvalues

To obtain specific values for energy, you operate on the wavefunction with the quantum mechanical operator associated with energy, which is called the Hamiltonian. The operator of the Hamiltonian on the wavefunction is the Schrodinger equation. Solutions exist for the time-independent Schr equation only for certain values of energy, and these values are called eigen values of energy

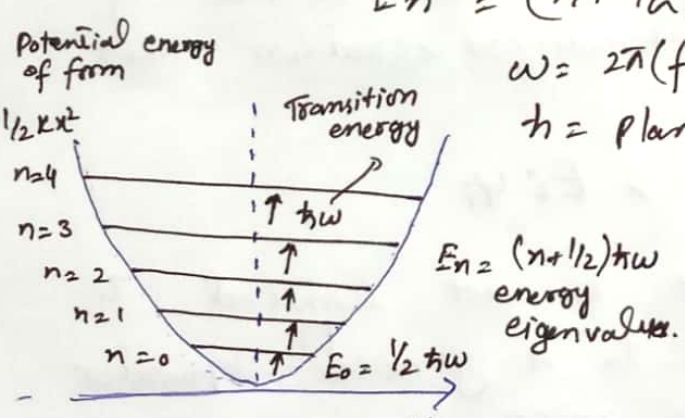
Example

The energy eigenvalues of the ~~Harmonic~~ Quantum Harmonic oscillator are given by

$$E_n = (n + 1/2)\hbar\omega \qquad n = 0, 1, 2, 3 \ldots$$

$\omega = 2\pi$ (frequency)

$\hbar$ = Planck's constant / 2π



The energy eigenvalues may be discrete for small values of energy, they usually become continuous at high enough energies because the system can no longer exist as a bound state. For a more realistic Harmonic oscillator potential, the energy eigenvalues get closer together as it approaches the dissociation energy. The energy levels after dissociation can take the continuous values associated with free particles.