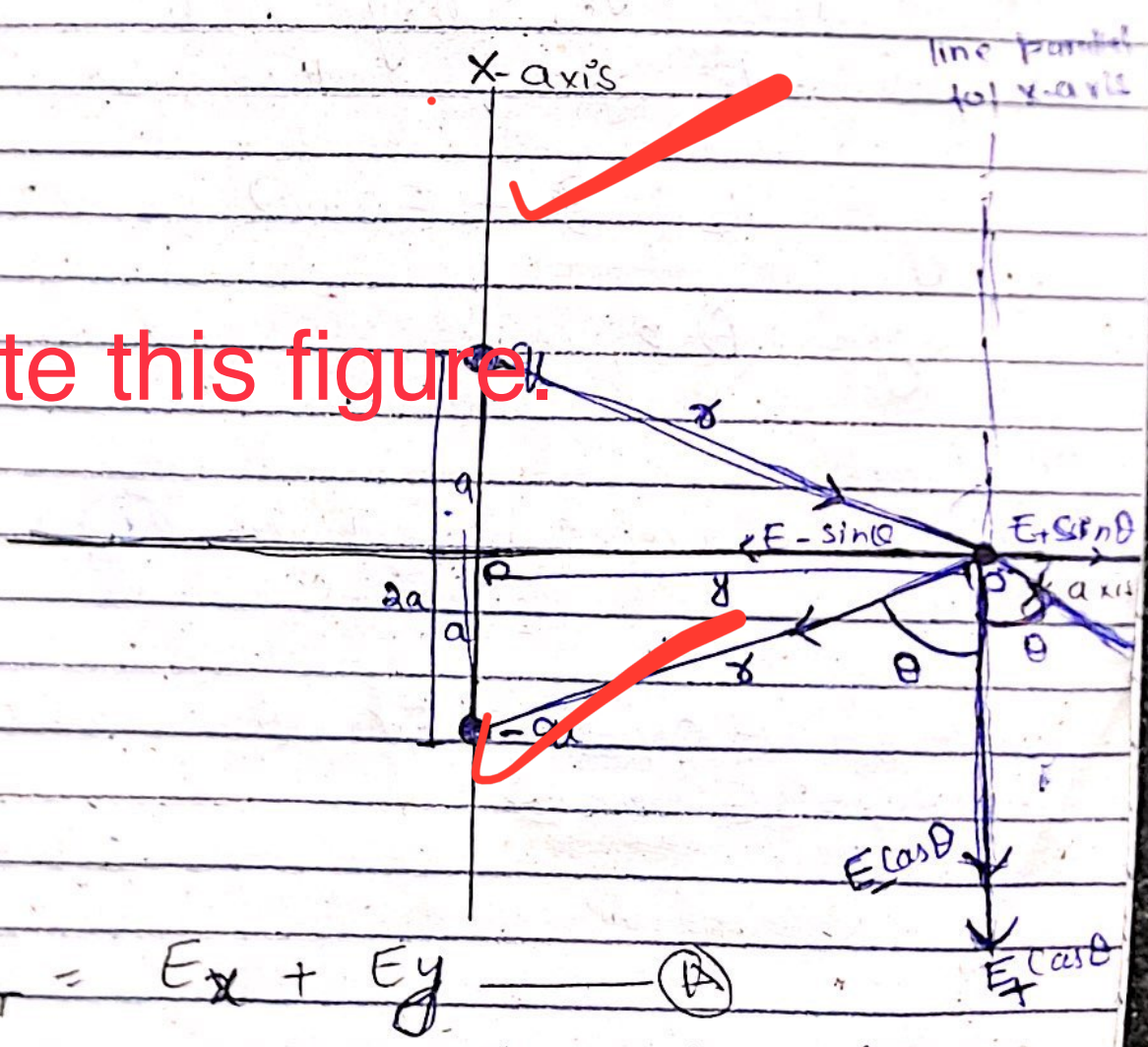
EMT PP -2022

(1) Given info

Positive charge = q
Negative charge = $-q$
Both charges equidistant from origin
Separation = $2a$
E at P on Y-axis = ?
$y \gg a$ & $\epsilon_0 = 8.85 \times 10^{-12}$

Sketching the dipole as per the given info



Elaborate this figure.

$E_T = E_x + E_y$ ——— (A)

Electric field along x-axis

$$E_x = E_+ \cos\theta + E_- \cos\theta \quad \text{---(1)}$$

Electric field along y-axis

$$E_y = E_+ \sin\theta + E_- \sin\theta$$

As evident from the figure that both $\sin\theta$ components are equal in magnitude but opposite in direction, so they will cancel-out each other, so

$$E_y = E\sin\theta - E\sin\theta$$

$$E_y = 0$$

Suppress the subscript in (1) to

$$E_x = E\cos\theta + E\cos\theta$$

$$E_x = 2E\cos\theta$$

Put in (A)

$$E_T = E_x + E_y$$

$$E_T = 2E\cos\theta + 0$$

$$E_T = 2E\cos\theta \quad \text{(B)}$$

Going for $\cos\theta$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{2a}{r}$$

Going for E

As we know

$$E = \frac{F}{q_0} = \frac{kqq_0}{q_0 r^2}$$

$$E = \frac{kq}{r^2}$$

Going for r^2

Putting Pythagorean theorem in figure

$$r^2 = y^2 + a^2$$

Putting this in place of r

$$E = \frac{kq}{y^2 + a^2}$$

And also

$$\cos\theta = \frac{2a}{\sqrt{y^2 + a^2}}$$

Putting both of the values in (E)

$$E_T = 2\left(\frac{kq}{y^2 + a^2}\right)\left(\frac{2a}{\sqrt{y^2 + a^2}}\right)$$

$$E_T = 2\left(\frac{1}{4\pi\epsilon_0}\frac{q}{y^2 + a^2}\right)\frac{2a}{\sqrt{y^2 + a^2}}$$

$$E_T = \frac{qa}{\pi\epsilon_0 (y^2 + a^2)^{3/2}}$$

As we know from the definition of dipole moment

$$\vec{P} = |q|d = qa$$

So

$$E_T = \frac{\bar{P}}{\pi \varepsilon_0 (y^2 + a^2)^{3/2}}$$

Now, as the condition is given that $y \gg a$, So a will be neglected.

$$E_T = \frac{\bar{P}}{\pi \varepsilon_0 (y^2)^{3/2}}$$

$$E_T = \frac{\bar{P}}{\pi \varepsilon_0 y^3}$$

This is the total electric field due to a dipole under given conditions.

(b) Given info

Source charge = Q

distance = x

Vector form of Electric field = $\vec{E}$ = ?

Graph of $\vec{E}$ = ?

From the definition of

$$E = \frac{F}{q_0} \quad \text{---(1)}$$

As we know

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Here

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q(q_0)}{r^2}$$

$$F = \frac{Qq_0}{4\pi\epsilon_0 r^2}$$

Put in (1)

$$E = \frac{Qq_0}{4\pi\epsilon_0 r^2 q_0}$$

So,

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

OR

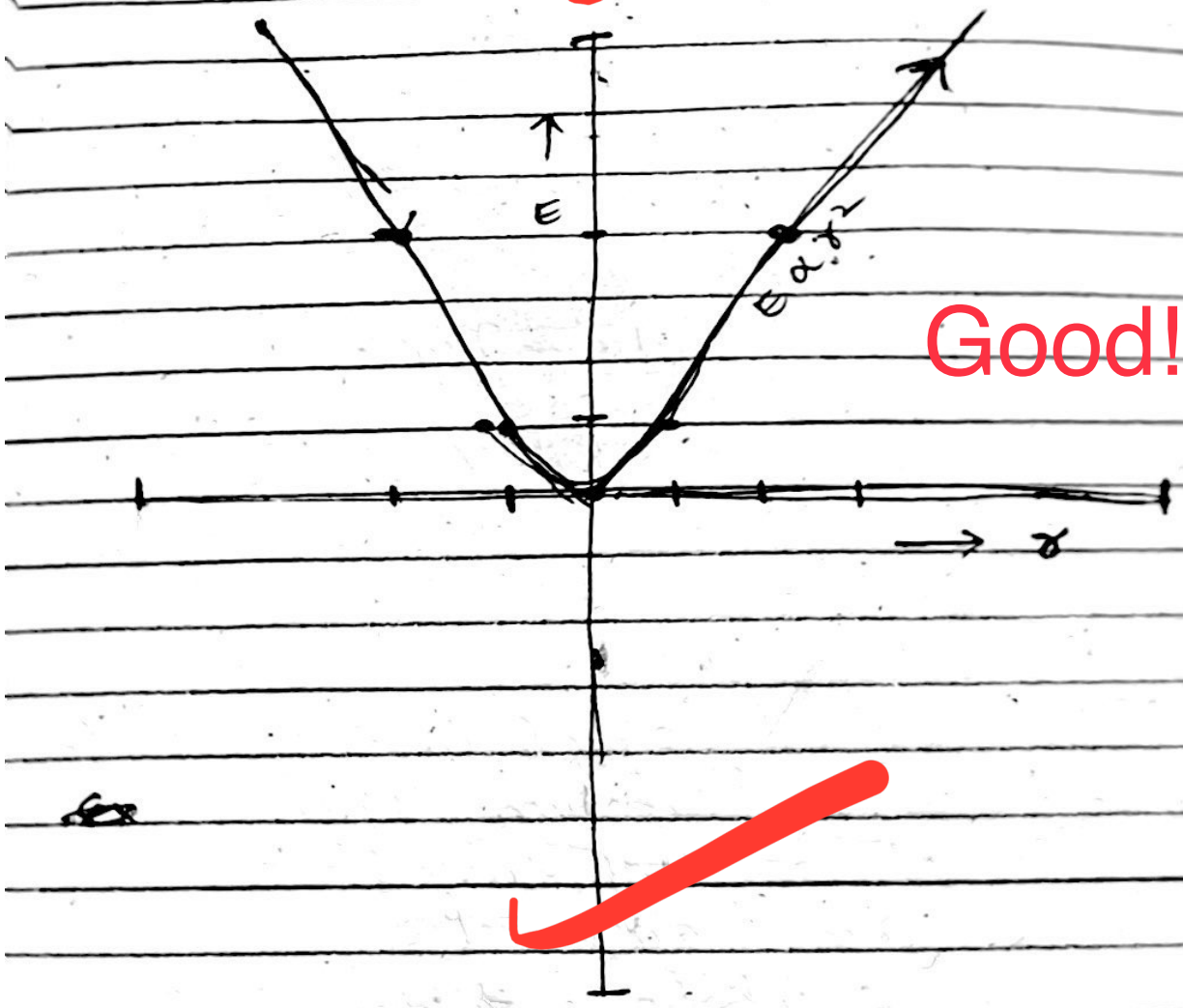
$$\boxed{E = k \frac{Q}{r^2}}$$

Graphical explanation

From the equation

$$E \propto \frac{1}{r^2}$$

Sketching E w.r.t r^2



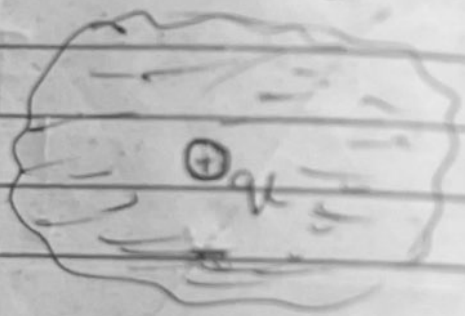
Good!

(c) Electric field and a dipole

Electric field:

A region surrounding the charge in which the charge exerts its electrostatic force on other charges is called the electric field. Mathematically

$$E = \frac{F}{q_0}$$



Where

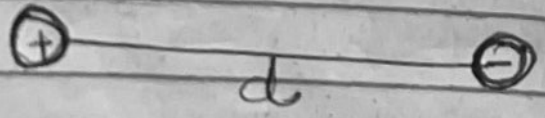
F = electrostatic force

q_0 = point charge

Derivation?

Dipale

Two charges equal in magnitude & opposite in direction when placed at a very small distance from each other constitute a dipole. Mathematically

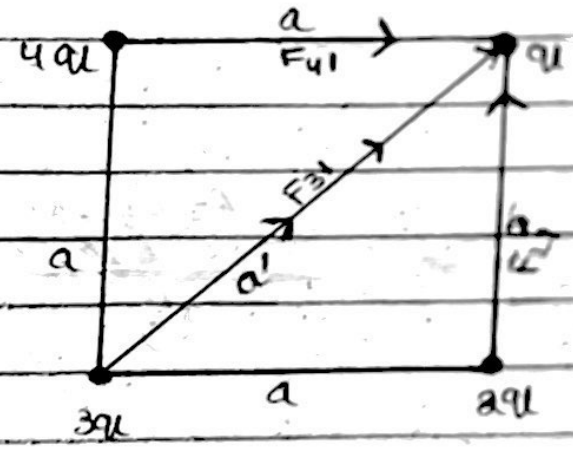


Q5(a)

Given info:

→ 4 Charges, counter clockwise arranged on corners of a square.
→ charge = $q, 2q, 3q, 4q$
→ Square's arm length = a
→ E-Field at q = ?
→ E Force at q = ?

Sketching the given condition.



Electric Force on q.

$$F_T = F_{21} + F_{31} + F_{41} \quad \text{---(1)}$$

$$F_{21} = K \frac{q_1 q_2}{r^2} = \frac{K(2q)(q)}{(a)^2} = \frac{2Kq^2}{a^2}$$

$$F_{41} = K \frac{q_1 q_2}{r^2} = \frac{K(4q)(q)}{a^2} = \frac{4Kq^2}{a^2}$$

For F_{31}, we have to calculate the $3q$'s distance from q, which is a'.

By Pathagorean theorem

$$hyp^2 = per^2 + base^2$$

$$(a')^2 = a^2 + a^2$$

$$(a')^2 = 2a^2$$

$$a' = a\sqrt{2}$$

So

$$F = \frac{k q_1 q_2}{r^2} = \frac{k(3q)(q)}{(a')^2} = \frac{3kq^2}{2a^2}$$

Putting all in (A)

$$F_T = \frac{2kq^2}{a^2} + \frac{4kq^2}{a^2} + \frac{3kq^2}{2a^2}$$

$$F_T = \frac{kq^2}{a^2}\left(2 + 4 + \frac{3}{2}\right)$$

$$F_T = \frac{kq^2}{a^2}\left(6 + \frac{3}{2}\right)$$

$$F_T = \frac{kq^2}{a^2}\left(\frac{12+3}{2}\right)$$

$$F_T = \frac{kq^2}{a^2}\left(\frac{15}{2}\right)$$

$$\boxed{F_T = \frac{15kq^2}{2a^2}}$$

Electric field at charge

We know that

$$E = \frac{F}{q_0} = \frac{F}{q}$$

$$E = \frac{15kq^2}{2a^2} \times \frac{1}{q}$$

$$\boxed{E = \frac{15kq}{2a^2}}$$

b) Given info:

→ Parallel plate capacitor
→ Space between plates = 1mm
→ Surface Area of each plate = ?
→ Capacitance = 1F
→ $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$

Soln:

$d = 1mm = 1 \times 10^{-3} m$
$C = 1F$
$A = ?$

We know that

$$C = \frac{A \varepsilon_0}{d}$$

$$A = \frac{Cd}{\varepsilon_0}$$

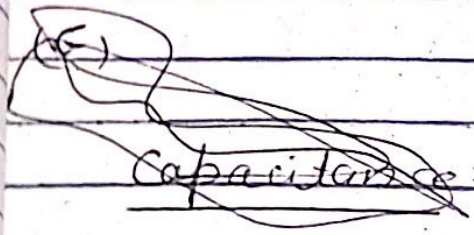
$$A = \frac{(1)(1 \times 10^{-3})}{8.85 \times 10^{-12}}$$

$$A = 1.13 \times 10^{8} m^2$$

∴ it possible to build it in b.

From the amount of the required area it is not possible or at least near

impossible to build a capacitor with such dimensions in a lab.



Capacitance:

(c) Given info:

Separation b/w || plates = $d = x$

di-electric const = $\varepsilon_r = \gamma$

Thickness = $T = \frac{1}{3}x$

Capacitance = ?

Formula

$$C = \frac{A \varepsilon_0 \varepsilon_r}{d}$$

Going for Area:

$$A = (d)(T)$$
$$= x\left(\frac{1}{3}x\right)$$
$$= \frac{1}{3}x^2$$

So

$$C = \frac{1/3\, x^2 (8.85 \times 10^{-12})(\gamma)}{x}$$

$$= \frac{(8.85 \times 10^{-12})(\gamma)(x)}{3}$$

So $C = \gamma \times 2.9 \times 10^{-12}$

Good!