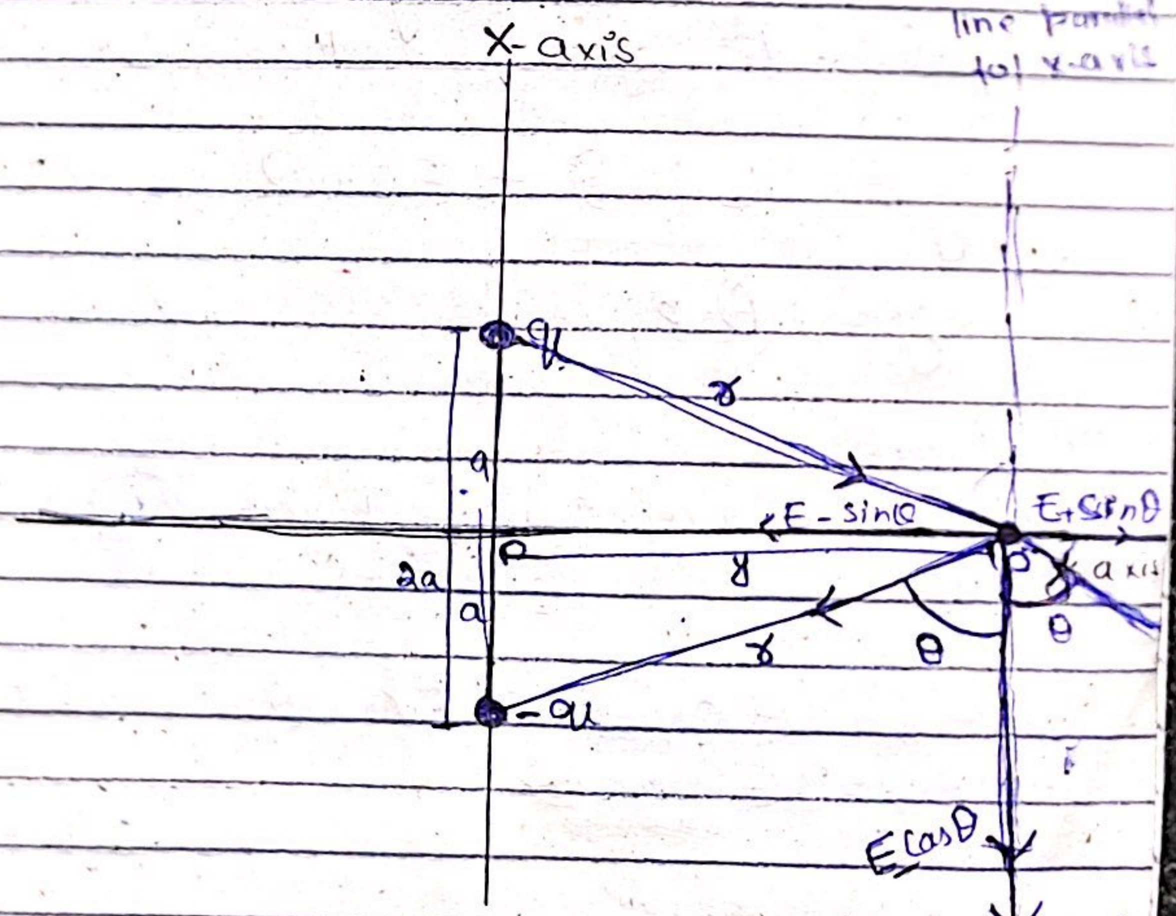
EMT PP - 2022

(1) Given info

Positive charge = q
Negative charge = $-q$
Both charges equidistant from origin
Separation = $2a$
E at P on y-axis = ?
$y >> a$ & $\varepsilon_0 = 8.85 \times 10^{-12}$

Sketching the dipole as per the given info



$$E_T = E_x + E_y \quad \text{——— (A)}$$

Electric field along x-axis

$E_x = E_+ \cos\theta + E_- \cos\theta$ — ①

Electric field along y-axis

$E_y = E_+ \sin\theta + E_- \sin\theta$

As evident from the figure that both $\sin\theta$ components are equal in magnitude but opposite in direction, so they will cancel-out each other, so

$E_y = E\sin\theta - E\sin\theta$

$E_y = 0$

~~This ab~~
Suppress the subscript in (1) too:

$E_x = E\cos\theta + E\cos\theta$

$E_x = 2E\cos\theta$

Put in A

$E_T = E_x + E_y$

$E_T = 2E\cos\theta + 0$

$E_T = 2E\cos\theta$ ——— B

Going for $\cos\theta$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{2a}{r}$$

Going for E

As we know

$$E = \frac{F}{q_0} = \frac{kqq_0}{q_0 r^2}$$

$$E = \frac{kq}{r^2}$$

Going for r^2

Putting Pythagorean theorem in figure

$$r^2 = y^2 + a^2$$

Putting this in place of r

$$E = \frac{kq}{y^2 + a^2}$$

And also

$$\cos\theta = \frac{2a}{\sqrt{y^2 + a^2}}$$

Putting both of the values in (E)

$$E_T = 2\left(\frac{kq}{y^2 + a^2}\right)\left(\frac{2a}{\sqrt{y^2 + a^2}}\right)$$

$$E_T = \cancel{2}\left(\frac{1}{\cancel{4}\pi\varepsilon_0}\frac{q}{y^2 + a^2}\right)\frac{\cancel{2}a}{\sqrt{y^2 + a^2}}$$

$$E_T = \frac{qa}{\pi\varepsilon_0 (y^2 + a^2)^{3/2}}$$

As we know from the definition of dipole moment

$$\bar{P} = |q|d = |q|a$$

So

$$E_T = \frac{\bar{P}}{\pi \varepsilon_0 (y^2 + a^2)^{3/2}}$$

Now, as the condition is given that $y >> a$, So a will be neglected

$$E_T = \frac{\bar{P}}{\pi \varepsilon_0 (y^2)^{3/2}}$$

$$\boxed{E_T = \frac{\bar{P}}{\pi \varepsilon_0 y^3}}$$

This is the total electric field due to a dipole under given conditions.

(b) <u>Given info</u>

Source charge $= Q$

distance $= r$

Vector form of electric field $= \vec{E} = ?$

Graph of $\vec{E} = ?$

From the definition of

$$E = \frac{F}{q_0} \quad \text{——— (1)}$$

As we know

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Here

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q(q_0)}{r^2}$$

$$F = \frac{Q q_0}{4\pi\epsilon_0 r^2}$$

Put in (1)

$$E = \frac{Q \cancel{q_0}}{4\pi\epsilon_0 r^2 \cancel{q_0}}$$

So,

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

OR

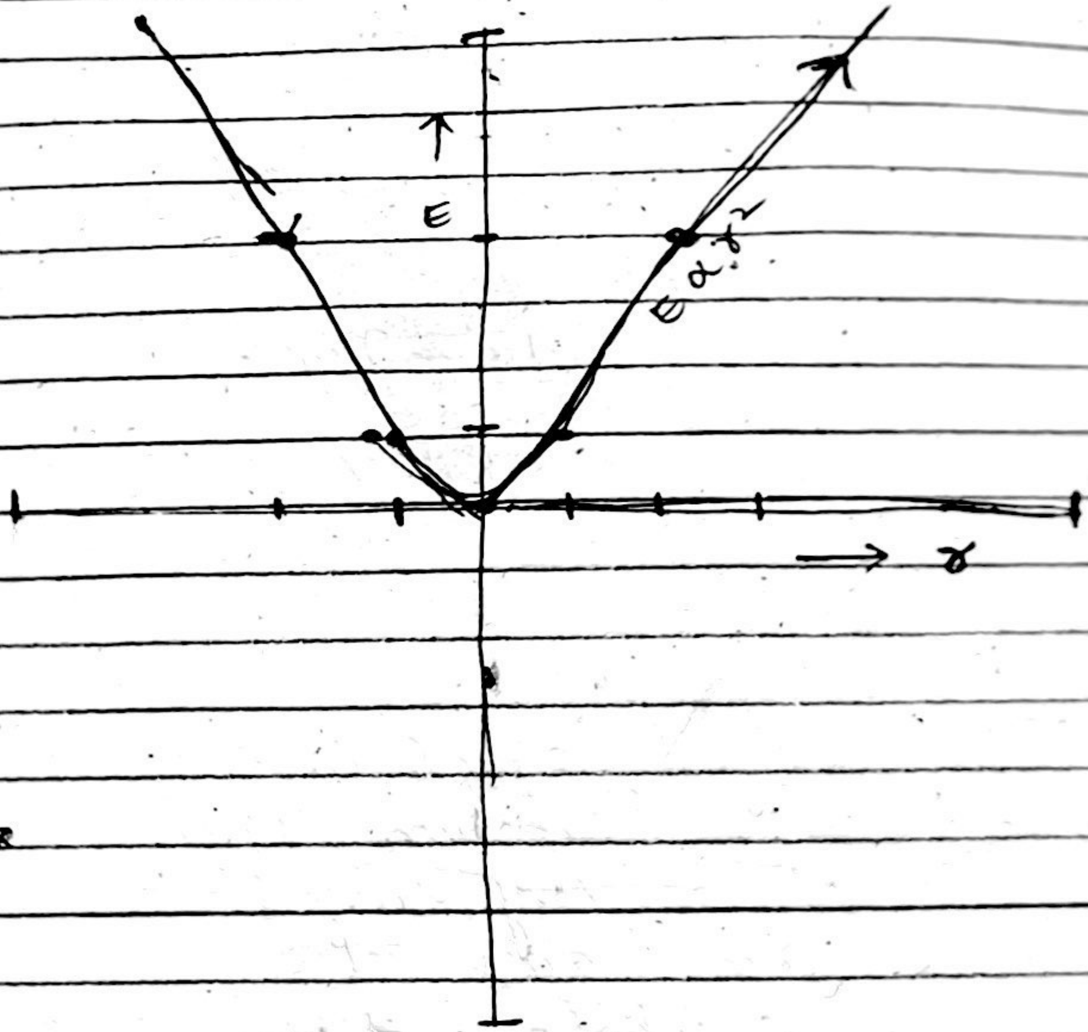
$$\boxed{E = k \frac{Q}{r^2}}$$

Graphical explanation

From the equation

$$E \propto \frac{1}{r^2}$$

Sketching E w.r.t r^2

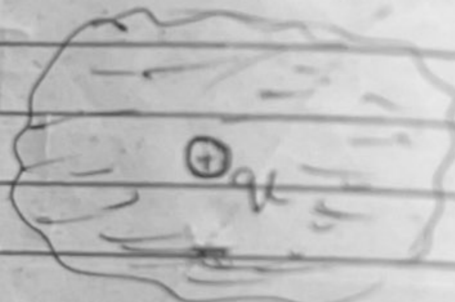


~~(c) Electric field and a dipole~~

Electric field:

A region surrounding the charge in which the charge exests its electrostatic force on other charges is called the electric field. Mathamatically

$$E = \frac{F}{q_0}$$



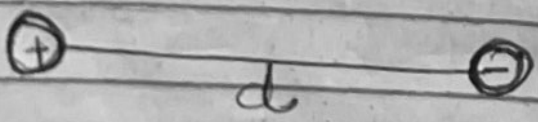
Where

F = electrostatic force

q_0 = point charge

Dipale

Two charges, equal in magnitude & opposite in direction when placed at a very small distance from each other, constitute a dipale. ~~Mathematically~~

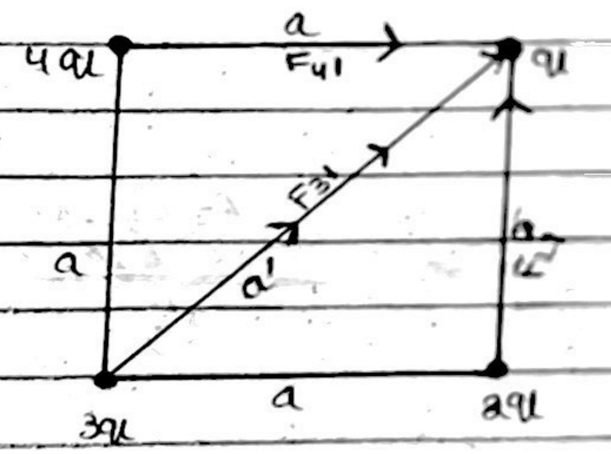


Q 5 (a)

Given info :

→ 4 Charges, counter clock-wise arranged on corners of a square
→ charge = q, $2q$, $3q$, $4q$
→ square's arm length = a
→ E. Field at q = ?
→ E Force at q = ?

Sketching the given condition :



Electric Force on q.

$$F_T = F_{21} + F_{31} + F_{41} \quad \text{—— (A)}$$

$$F_{21} = k \frac{q_1 q_2}{r^2} = \frac{k(2q)(q)}{(a)^2} = \frac{2kq^2}{a^2}$$

$$F_{41} = k \frac{q_1 q_2}{r^2} = \frac{k(4q)(q)}{a^2} = \frac{4kq^2}{a^2}$$

For F_{31}, we have to calculate the $3q_1$'s distance from q_1, which is a'.

By Pathagoream theorem

$$hyp^2 = per^2 + base^2$$

$$(a')^2 = a^2 + a^2$$

$$(a')^2 = 2a^2$$

$$a' = a\sqrt{2}$$

So

$$F = \frac{k q_1 q_2}{\cancel{r}^2} = \frac{k(3q_1)(q_1)}{(a')^2} = \frac{3kq_1^2}{2a^2}$$

Putting all in (A)

$$F_T = \frac{2kq_1^2}{a^2} + \frac{4kq_1^2}{a^2} + \frac{3kq_1^2}{2a^2}$$

$$F_T = \frac{kq_1^2}{a^2}\left(2 + 4 + \frac{3}{2}\right)$$

$$F_T = \frac{kq_1^2}{a^2}\left(6 + \frac{3}{2}\right)$$

$$F_T = \frac{kq^2}{a^2}\left(\frac{12+3}{2}\right)$$

$$F_T = \frac{kq^2}{a^2}\left(\frac{15}{2}\right)$$

$$\boxed{F_T = \frac{15kq^2}{2a^2}}$$

Electric field at charge

We know that

$$E = \frac{F}{q_0} = \frac{F}{q}$$

$$E = \frac{15kq^{\cancel{2}}}{2a^2} \times \frac{1}{\cancel{q}}$$

$$\boxed{E = \frac{15kq}{2a^2}}$$

(b) Given info:

→ Parallel plate capacitor
→ Space between plates = 1mm
→ Surface Area of each plate = ?
→ Capacitance = 1F
→ $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Soln:

$d = 1mm = 1 \times 10^{-3} m$
$C = 1F$
$A = ?$

We know that

$$C = \frac{A\varepsilon_0}{d}$$

$$A = \frac{Cd}{\varepsilon_0}$$

$$A = \frac{(1)(1 \times 10^{-3})}{8.85 \times 10^{-12}}$$

$$A = 1.13 \times 10^{8} m^2$$

..t possible to build it in

b.

From the amount of the ..quired Area, it is not ..ossible or at least near impossible to build a capacitor with such dimensions in a lab.

~~(c) Capacitance:~~

(c) Given info:

Separation b/w || plates $= d = x$

di-electric const $= \varepsilon_r = \gamma$

Thickness $= T = \frac{1}{3}x$

Capacitance $= ?$

Formula

$$C = \frac{A \varepsilon_o \varepsilon_r}{d}$$

Going for Area:

$$A = (d)(T)$$

$$= x\left(\frac{1}{3}x\right)$$

$$= \frac{1}{3}x^2$$

So

$$C = \frac{1/3\, x^{\cancel{2}} (8.85 \times 10^{-12})(\gamma)}{\cancel{x}}$$

$$= \frac{(8.85\times10^{-12})(\gamma)(x)}{3}$$

So $$\boxed{C = \gamma x\,2.95\times10^{-12}}$$