Lecture # 01.

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What we will study:

1. Basic Arithmetic
   - mathematical operators.
   - powers
   - Rounding of numbers.
   - number systems.
   - Decimal multiplication
   - Fractions
   - LCM & HCF
2. percentage.
3. Sets and sets Theory.
4. Algebra
   - Age problems
   - Word problems / linear algebraic equation
   - Basic algebraic arithmetic
5. Geometry.
6. Ratio and proportion
7. ~~Probability~~
   Statistics
   - mean
   - mode
   - Range
   - probability.
8. Logical and analytical reasoning / ability
   - Analytical reasoning
   - Blood Relations
   - Series.

Mathematical operators:

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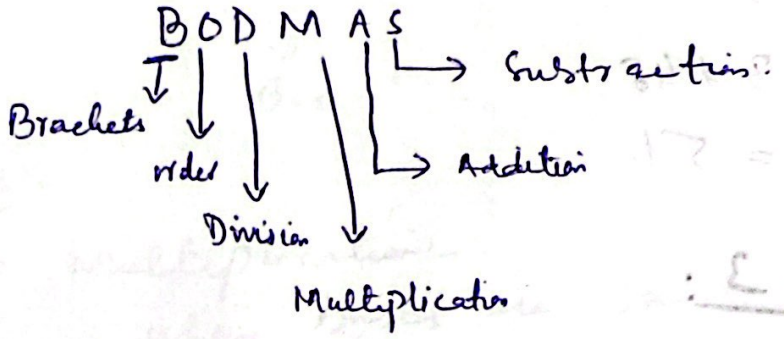
+, −, ×, ÷

Using BODMAS



Example ①:

$$4(10+15\div5\times4-2+2)$$

Step ① Brackets

Step ② Division

$$4(10+3\times4-2+2)$$

Step ③ Multiplication

$$4(10+12-2+2)$$

Step ④ Addition:

$$4(22-4)$$

Step ⑤ Subtraction

$$4(18)$$

Answer = $4\times18=72$.

Exaple 2:

$$3+2^4 \times (15 \div 5)$$

$$3+2^4 \times (3)$$

$$3 + 2 \cdot 2 \cdot 2 \cdot 2 \times 3$$

$$3 + 16 \times 3$$

$$3 + 48$$

$$= 51.$$

Exaple 3:

$$14+26-27 \div 3 \times 2$$

① Division:

$$14+26-9 \times 2$$

② Multiplication:

$$14+26-18$$

③ Addition:

$$40-18$$

④ Subtraction:

$$22$$

Answer = 22.

Brackets

( ) → round brackets/ parenthesis

{ } → curly brackets

[ ] → square brackets

owers / Exponents:

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A mathematical notation that represents the number of times a base should be multiplied by itself. e.g

7^{3} → ③ → exponent.
↓
Base

Rules:-

(A) Multiplication:-
when bases are same, powers are added. e.g

$$x^2 \cdot x^3 = x^{2+3} = x^5.$$

when base are different, exponents cannot be added. e.g

$$a^2 \cdot b^5 = a^2 b^5.$$

(B) power raised to the power:-
when a power is raised to the power, they are inter multiplied.
e.g

$$(a^b)^c = a^{b \times c} = a^{bc}.$$

$$(2^2)^3 = 2^{2\times3} = 2^6 = 2\cdot2\cdot2\cdot2\cdot2\cdot2 = 64$$

(3) Fraction form:-
(a) negative power:

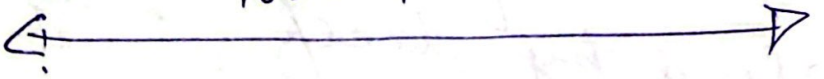
$$a^{-2} = \frac{1}{a^2}.$$

$$\frac{x^{-a}}{x^{-b}} = \frac{x^b}{x^a}.$$

④ Zero Exponent:

Any number that has an exponent of 0 is equal to 1. e.g

$100^0 = 1$



⑤ $\sqrt{} = \frac{1}{2}$ Square root

$\sqrt[3]{} = \frac{1}{3}$ cube root

$\sqrt{4} = 2.$

365
100 10 unit

365.3479 → to
100 10 unit
10 100
1/10 1/1000

Rounding of numbers:

Rules: (i) Locate the underline digit of the decimal place which needs to be rounded off.

(ii) Consider the digit to the right of the underlined digit.

(iii) If the digit = 5
> 5
e.g 6, 7, 8, ...
the underlin digit is increased by 1.

(iv) If the underlin digit < 5, e.g 4, 3, 2, 1, etc. then keep the underlin unchanged.

Rounding of whole no.
87
90

e.g

(1) 72.36 (nearest 10th)
72.4

73.5 (odd)
74 (↑)
68.5 (even)
68

(2) 72.84
72.8

714.542 (nearest 100th)
714.54

714.545 (nearest 100th)
714.55

8.2342 (1000th
8.234
8.2437
8.244

$2.746 \rightarrow 2.75$ (↑)

$2.743 \rightarrow 2.74$ (↓)

2.745

↓

Preceding no. is even $\rightarrow 2.74$

$2.735 \rightarrow 2.74$

odd → up

even → do not change

726.835 (upto 2 decimal

Preceding number is 3 odd

726.84 (up by 1)

24.8514

24.8

① Additive inverse:

It means changing the sign of the number and adding it to the original number to get the answer 0.

$x, -x$

$x - x = 0$

② Multiplicative inverse

A number when multiplied by itself yields the multiplicative identity 1

$\Rightarrow$ Multiplicative identity $= 1$

Additive identity $= 0$

Basic mathematics:

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Number System:

(1) Natural numbers:

Natural numbers are defined as the counting numbers starting with 1 and increasing by 1 forever.

$$N = \{1, 2, 3, 4, 5, \dots\}.$$

(2) Whole numbers:

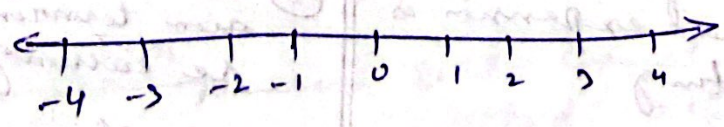
It is the set of natural numbers and Zero.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}.$$

(3) Integers:-

An integers is a number that has no fraction or decimal. It includes Zero, natural numbers and their additive inverse.

$$Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$$



(4) Rational numbers:-

Numbers that can be written in the form of $\frac{p}{q}$, where $q \neq 0$ are called rational numbers. e.g

$$\frac{4}{7}, \frac{3}{2}, \frac{5}{8},$$

It is denoted by Q.

The decimal expression of a rational number is terminating or non-terminating recurring.

(5) Irrational numbers:-

Numbers that cannot be written in the form of $\frac{p}{q}$ (or in the form of fraction) are called irrational numbers.

e.g

$$\sqrt{5}, \sqrt{10}, \sqrt{3}, \sqrt{15}.$$

It is denoted by Q'.

Rational	Irrational
(1) All solvable square roots are rational. e.g $\sqrt{25} = 5$	(1) All insolvable square roots are irrational. e.g $\sqrt{5}$,
(2) The decimal expansion is terminating or non terminating; recurring	(2) The decimal expansion is non terminating non recurring.
(3) Example: 0.666, 0.757575...	(3) Example $\sqrt{2}$ etc.

(∴) Pi, π looks like rational $\frac{22}{7}$ but it is irrational because non-terminating / non recurring = 3.14 2857

⑥ Real numbers:

It is the union of both rational and irrational numbers. They can be both positive or negative. It is denoted by "R". It includes all natural numbers, decimals and fractions.

e.g

3, 0, 1.5, $\frac{3}{2}$, $\sqrt{5}$.

⑦ Complex numbers:

A non real number that can be written in the form of $a+bi$.

$a + bi$

↓ ↓

a represents the real part

b represents the imaginary part.

$x^2 + 1 = 0$

$x^2 = -1$

$x = \sqrt{-1}$

$i = \sqrt{-1}$

It is denoted by z.

⑧ Even numbers:

Numbers that has no remainder when divided by 2. Or number that are completely divisible by 2.

e.g

0, 2, 4, 6, 8, ...

⑨ Odd numbers:

Numbers that cannot be completely divided by 2 are called odd numbers.

e.g

1, 3, 5, 7, ...

(10) Prime numbers:

A positive integer / number th
can be divided by 1 and itself. e.g
2, 3, 5, 7, 13, 23.

(11) Composite numbers:

positive numbers that has
factors other than 1 and itself.

OR

numbers that are not prime are called
composite numbers. e.g

4 = 1, 2, 4
8 = 1, 2, 4, 8
24 = 1, 2, 3, 4, 6, 8, 12, 24

Dealing with Decimal numbers.

(1) Addition of Decimals:

$12.5 + 6.23$

$$\begin{array}{r} 12.5 \\ 6.23 \\ \hline 18.73 \end{array}$$

$$\begin{array}{r} 12.5 \\ 6.23 \\ \hline 7.40 \end{array}$$

(ii) $14.7 + 3.685$

$$\begin{array}{r} 14.7 \\ 3.685 \\ \hline 18.385 \end{array}$$

(iii) $3.82 + 13.6 + 5.431$

$$\begin{array}{r} 3.820 \\ 13.600 \\ 5.431 \\ \hline 22.851 \end{array}$$

Subtraction of Decimals:-

(1) $8.5 - 6.23$

$$\begin{array}{r} 8.5^{\prime}0 \\ 6.23 \\ \hline 2.27 \end{array}$$

(2) $7.5 - 4.368$

$$\begin{array}{r} {}^{4} \\ 7.5^{9}0^{10}8 \\ 4.368 \\ \hline 3.132 \end{array}$$

(3) $5.3 - 2.8$

$$\begin{array}{r} 5.^{\prime}3 \\ 2.8 \\ \hline 2.5 \end{array}$$

(4) $9.6 - 4.17$

$$\begin{array}{r} 9.60 \\ 4.17 \\ \hline 5.43 \end{array}$$

Subtracting whole no. from decimals.

(5) $8.3 - 5$

$$\begin{array}{r} 8.3 \\ 5.0 \\ \hline 3.3 \end{array}$$

Subtracting decimal from whole no.

(6) $9 - 4.2$

$$\begin{array}{r} 9.^{\prime}0 \\ 4.2 \\ \hline 4.8 \end{array}$$

Decimal multiplication:

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① 6.4×3.1

First multiply both numbers without decimal.

$$\begin{array}{r} 64 \\ \times\ 31 \\ \hline 64 \\ 192\ 0 \\ \hline 1984 \end{array}$$

$6.4 \rightarrow$ (1) point
$3.1 \rightarrow$ (1) point from right side

Add both point and place the decimal accordingly.

$19.84 \rightarrow$ 2 points

② 11.3×2.5

$$\begin{array}{r} 113 \\ \times\ 25 \\ \hline 565 \\ 2260 \\ \hline 28.25 \end{array}$$

$\Rightarrow 28.25$.

How many digits are behind the decimal.

(iii)

1·25

5·2:

$$\begin{array}{r} 1{\cdot}25 \\ 5{\cdot}2 \\ \hline 250 \\ 6100 \\ \hline 6350 \end{array}$$

6·35.

(iv)

0·9

8·7

7·83

$$\begin{array}{r} 87 \\ 9 \quad 6 \\ \hline 783 \end{array}$$

$9 \times 7 = 63$

$9 \times 8 = 72$

Division of Decimals:

① $8.5 \div 3.4$

Divisor $3.4 \overline{)8.5}$ → Dividend

$34 \overline{)85}$ (Remove Decimals.

$$\begin{array}{r} 2.5 \\ 34 \overline{)85.0} \\ 68 \\ \hline 170 \\ 170 \\ \hline \end{array}$$

$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \end{array}^{2}$$

$\boxed{2.5}$ → Ans.

② $10.8 \div 1.5$

$$\begin{array}{r} 7.02 \\ 15 \overline{)108} \\ 105 \\ \hline 3.0 \\ 3.0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 15 \\ \times\ 7 \\ \hline 105 \end{array}^{3}$$

7.02

(iii) $9.01 \div 1.7$

$$17 \overline{)90.1} \quad 5.3$$

(more decimal 1 time)

85

5.1

5.1

0

$17 \times 5 = 85$

$17 \times 3 = 51$

(5.3) → Answer.

(iv) $9.66 \div 2.3$

$$23 \overline{)96.6} \quad 4.2$$

92

4.6

4.6

0

$23 \times 4 = 92$

$23 \times 2 = 46$

$23 \times 2 = 4.6$

(4.2) → Answer.

LCM & HCF

LCM = Least Common multiple
HCF = Highest Common factor.

(1)

HCF

28, 70, 84

2 | 28
2 | 14
7 | 7
1	

2 | 70
7 | 35
5 | 5
1	

2 | 84
2 | 42
7 | 21
3 | 3
1	

$28 = 2 \times 2 \times 7$
$70 = 2 \times 5 \times 7$
$84 = 2 \times 2 \times 7 \times 3$

HCF $= 2 \times 7 = 14$

LCM

28, 70, 84

2 | 28
2 | 14
7 | 7
1	

2 | 70
5 | 35
7 | 7
1	

2 | 84
2 | 42
3 | 21
7 | 7
1	

$28 = 2 \times 2 \times 7$
$70 = 2 \times 5 \times 7$
$84 = 2 \times 2 \times 3 \times 7$

LCM $= 2 \times 7 \times 2 \times 5 \times 3$
$= 420$

(2) Find the HCF & LCM of 24, 36

2 | 24
2 | 12
3 | 6
2 | 2
1	

2 | 36
2 | 18
3 | 9
3 | 3
1	

$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$

HCF $= 2 \times 2 \times 3 = 12$

LCM $= 2 \times 2 \times 3 \times 2 \times 3$
$= 72.$

Q: Find the greatest number that divides 204 and 64 without a remainder.

Sol:

2	204
2	102
3	51
17	17
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

$204 = 2 \times 2 \times 3 \times 17$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$HCF = 2 \times 2 = 4$

Division method.

3

64 / 204

192 | 51

12 | 64

60 | 3

4 | 12

12

0

HCF ← 4

Q: What is the least number of children who may be arranged in rows of 12, 14 or 16, 18 in each row:

Sol:-

2	12
2	6
3	3
	1

2	14
7	7
	1

2	16
2	8
2	4
2	2
	1

2	18
3	9
3	3
	1

$12 = 2 \times 2 \times 3$

$14 = 2 \times 7$

$16 = 2 \times 2 \times 2 \times 2$

$2 \times 2 \times 3 = 12$

$2 \times 7 = 14$

$2 \times 2 \times 2 \times 2 = 16$

$12 = 2 \times 2 \times 3$

$16 = 2 \times 2 \times 2 \times 2$

$18 = 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 336$

$2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2$

336 is the least no. of children who may be arranged in rows of 12, 14 & 16.

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Fractions :

A small amount or a portion of something.

e.g

$\frac{1}{2}$ → two equal part of something.

$\frac{1}{3}$ → three equal parts of something.

$\frac{1}{3}$ → numerator

$\frac{1}{3}$ → denominator.

Example:- Division of a pizza into 8 slices

$\frac{1}{8}$ → Nu
8 → Den

Types:

① Proper fraction:

A Fraction who numerator is less than the denominator.. e.g

$\frac{4}{5}$, $\frac{3}{7}$ etc.

② Improper fraction:

A fraction who numerator is greater than or equal to the denominator.

e.g

$\frac{4}{3}$, $\frac{3}{2}$, $\frac{10}{5}$, $\frac{15}{15}$, etc.

③ Mixed fraction:-

A B.number consisting of a whole number and a proper fraction.

$a\ \frac{b}{c}$ → proper fraction.

whole number

$1\frac{1}{2} = \frac{3}{2}$.

$2\frac{3}{4} = \frac{11}{4}$

$12\frac{2}{3} = \frac{38}{2}$.

Operations on fractions:

(1) Addition:

$\frac{1}{16} + \frac{3}{16} + \frac{5}{16} = \frac{1+3+5}{16} = \frac{9}{16}$

$\frac{1}{2} + \frac{3}{5} + \frac{7}{3}$

$\frac{15+18+70}{30}$

(2) Subtraction:-

$\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

$\frac{1}{2} - \frac{3}{7}$

$\frac{7-6}{14} = \frac{1}{4}$

(3) Multiplication:-

$\frac{5}{3} \times \frac{2}{4} = \frac{10}{12}$.

(4) Division:-

Divide $\frac{1}{6}$ by $\frac{1}{3}$

$$\frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{6} \times \frac{3}{1} = \frac{3}{6_2} = \frac{1}{2}$$