General Ability Notes for CSS Candidates

NOA Academy, Islamabad

Table of Contents

Chapter 1: Basic Mathematical Skills

Mathematics: The branch of science which deals with numbers, equation functions and geometrical shapes.

Algebra: The part of mathematics in which letters and other symbols are used represent numbers in equations and formulae (plural of formula).

Mathematical operations:

The mathematical operations like addition, subtraction, multiplication and divisio are carried out in following order

1. Bracket
2. Division
3. Multiplication
4. Addition
5. Subtraction

You may remember it by

BDMAS

Example 1:

2+2×2=8 (incorrect)

2+2×2=6 (correct)

Example 2

Solve 2+3×2÷2+(5-7)

=2+3×2÷2+(-2)

=2+3×2÷2-2

=2+3×1-2

=2+3-2

= 5-2

= 3

Order of division of four and three numbers

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$$

$$\frac{\frac{a}{b}}{c} = \frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$$

Powers

1. In multiplication when bases are same, powers are added

$$x^a . x^b = x^{a+b}$$

Also

$$x^2 . x^5 = x^{2+5} = x^7$$

2. When there is power on a power of any variable, then powers of variable are simply multiplied

$$(x^a)^b = x^{ab}$$

Also

$$(x^2)^3 = x^6$$

3. When variable is moved from numerator to denominator or from denominator to nominator ,signs of the power are changed

$$\frac{x^a}{y^b} = \frac{y^{-b}}{x^{-a}}$$

Also

$$\frac{x^4}{x^3} = x^4 . x^{-3}$$

$$x^{4+(-3)}$$

$$x^{4-3}$$

$$x^1$$

$$x$$

4. The value of root is ½

$$\sqrt{x} = (x)^{\frac{1}{2}}$$

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$$\sqrt[3]{x} = (x)^{\frac{1}{3}}$$

$$\sqrt[4]{x} = (x)^{\frac{1}{4}}$$

Similarly in general

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

Fractions

$\frac{1}{2}$ is a fraction means two equal parts of anything.

e.g. $\frac{1}{2}$ *of a box is*

a.

1/2	1/2

$\frac{1}{3}$ *is a fraction which means three equal parts of anything*

b.

1/3	1/3	1/3

Boxes a and b clearly indicates that when denominator goes on increasing part of box become smaller.Hance greater the denominator smaller will be answer

Similarly

$$\frac{1}{\infty(infinity)} = 0$$

And

$$\frac{1}{0} = \infty(infinity)$$

Convert into simple fraction $5\frac{1}{7}$

$$5\frac{1}{7}$$

Multiply 5 and 7 and then add 1

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$$\frac{35+1}{7}$$

$$\frac{36}{7}$$

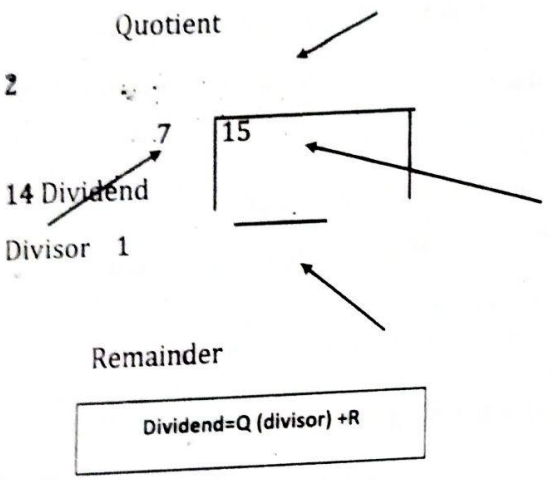
Similarly

$$2\frac{1}{7}$$

$$\frac{14+1}{7}$$

$$\frac{15}{7}$$

It can be expressed as



Dividend=Q (divisor) +R

How to decide which fraction is larger in case of difficult fractions.

- Make denominators of all fractions same by multiplying with a suitable number, in denominator. When you multiply the denominator, you have to multiply the numerator with same numbers so that original fraction does not change.

When one multiplies numerator or denominator only then mathematically incorrect.

- Now check which numerator is larger ,that fraction will be larger.

e.g. which fraction is larger?

$$\frac{2}{7}, \frac{4}{3}, \frac{1}{7}$$

$$\frac{3\times 2}{3\times 7}, \frac{7\times 4}{7\times 3}, \frac{3\times 1}{3\times 7}$$

$$\frac{6}{21}, \frac{28}{21}, \frac{3}{21}$$

So

$\frac{4}{3}$ *is larger fraction* (*as* 28 *is larger numerator*)

Basic mathematical formulae

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 - b^2 - 2ab$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$a^2 - b^2 = (a-b)(a+b)$$

Proof (if you forget)

$$(\boldsymbol{a+b})^2 = (\boldsymbol{a+b})(\boldsymbol{a+b})$$

Bases are same powers are added, simple multiplication

$$= a^2 + ab + ba + b^2$$

$$= a^2 + ab + ab + b^2$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

Similarly you can prove others.

Chapter 2: Number Theory

Natural numbers = N= {1,2,3,.....}

Whole numbers =W= {0,1,2,3,.....}

Integers =Z/I = {0,±1, ±2, ±3, ...}

Real numbers =ℝ= Large,small,positive,negative,fractions,whole numbers or decimal numbers are called real numbers e.g. 1,15,15.82,-0.1, ¾ etc.

Complex numbers = C=sum of real numbers and imaginary numbers i.e.

C= ℝ +Imaginary

..Normally expressed as C=a+ib

Where a=real part

b=imaginary part

(iota) $i=\sqrt{-1}$

Importance of imaginary numbers

Mathematically iota (i) or $\sqrt{-1}$ may appear in solving the equations like

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$\sqrt{x^2} = \pm\sqrt{-1}$$
$$x = \pm\sqrt{-1}$$
$$x = \pm i$$

Physically iota is used in quantum world (in which particles like electron, proton and their motion is discussed).specially the motion of electron around the nucleus in an atom is described by equation involving "i" (iota).

Rational numbers = Q=All numbers which can be expressed in $\frac{p}{q}$ form provided that q not equal to 0 (because $\frac{p}{0}$ will be equal to infinity)

Irrational numbers =Q'= All numbers which cannot be expressed as $\frac{p}{q}$ or q=0 are irrational.

Other distinctions between Q and Q'

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On the basis of square root (√)

Solvable square roots are rational

e.g.

$$\frac{\sqrt{9}}{2} = \frac{3}{2}$$

Insolvable square roots are irrational.

Q′="√ " involving fractions are often Q′

On the basis of decimal

Decimal

Terminating

e.g. 0.15 $= \frac{15}{100}$ (p/q) so rational

non-Terminating

Non-terminating & recurring

e.g. 0.131313..... (Rational)

non-terminating & non-recurring

e.g. 0.1312879..... (Irrational)

$\pi \approx \frac{22}{7}$ looks like rational as expressed in $\frac{p}{q}$ form but $\frac{22}{7} \approx 3.142857$ .....

So, it is irrational.

Prime numbers =P=A prime number is a whole number greater than 1 wh are divisible by 1 and itself.

e.g. { 2,3,5,7,11,13,17,19,23,29,.....}

Composite numbers = The numbers which are not prime, are called composite numbers.

Chapter 3: Percentage

Basic formulae:

1.

$$\text{percentage} = \frac{Given\ amount}{Total\ amount} \times 100$$

OR

$$\text{percentage} = \frac{part}{whole} \times 100$$

2.

$$\text{percentage increase} = \frac{increase\ in\ value}{original\ value} \times 100$$

$$\text{percentage Decrease} = \frac{decrease\ in\ value}{original\ value} \times 100$$

Trick

Keep in mind the symbol for:

Per means /

Cent means 100

Of means ( )

Percentage

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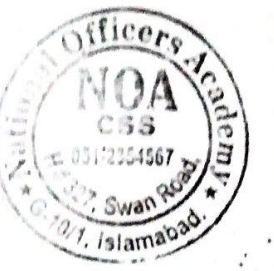
Percentage is derived from the Greek word **per centum** which means hundred



Question: Change into common fractions:

$$60\% = \frac{60}{100}$$

$$70\% = \frac{70}{100}$$

Question: Change into percentage:

$$\frac{70}{20}$$

$$\frac{70}{20} \times 100 = 35\%$$

Question: Change $45\frac{1}{2}\%$ into simple fraction:

$$= 45\frac{1}{2}\%$$

$$= \frac{91}{2}\%$$

$$= \frac{\frac{91}{2}}{100}$$

$$= \frac{91}{200}$$

Question: Find 25% of 500%:

$$= \frac{25}{100}(500)$$

$$= 125$$

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Example 1

In a parking there are 800 cars in which 80% cars are Pakistani made. Find the no. of Pakistani cars.

Sol:

$$Number\ of\ Pakistani\ cars = 80\%(800)$$

$$= \frac{80}{100}(800)$$

$$= 640\ cars$$

Example 2

In an aeroplane 400 passengers are board, in which 52% are Pakistani, 17% chines, 12% Iranian and rest of passengers are British.

1. Find the passengers of each country.

2. What is the percentage of British people?

Sol:

Pakistani passengers =

$$52\%(400)$$

$$= \frac{52}{100}(400)$$

$$= 52(4)$$

$$= 208$$

Chinese passengers =

$$= \frac{17}{100}(400)$$

$$= 68$$

British passengers =

$$52\% + 17\% + 12\% = 81\%$$

$$\text{British} = (100 - 81)\% = 19\%$$

Example 3

If $\frac{1}{8}$ is decreased by 25%, we get?

Sol:

25% of $\frac{1}{8}$ which is to be subtracted from $\frac{1}{8}$

$$= \frac{25}{100}(1/8)$$

$$= \frac{25 \times 1}{100 \times 8}$$

$$= \frac{1}{32}$$

Now

$$= \frac{1}{8} - \frac{1}{32}$$

$$= \frac{4 - 1}{32}$$

$$= \frac{3}{32}$$

$$= 0.093$$

Example 4

When 60 is subtracted from 60% of a number, the resulting no. is 60, what is the number?

Sol

Let the no. be x

60% of a no.

$\left(\frac{60}{100}\right)x$

According to given statement

$$\frac{6x}{10} - 60 = 60$$

$$\frac{6x}{10} = 60 + 60$$

$$\frac{6x}{10} = 120$$

$$6x = 1200$$

$$x = \frac{1200}{6}$$

$$x = 200$$

So the no is 200

Example 5

A candidate who gets 30% of total votes polled, is defeated by 15000 votes. Find the no.of votes of the winning candidate.

Sol:

Let total no. of votes polled=x

$$Defeated\ candidate's\ votes = \frac{30}{100}(x)$$

$$= \frac{3}{10}(x)$$

$$\text{Winnig candidate's votes} = x - \frac{3}{10}(x)$$

$$= \frac{10x - 3x}{10}$$

$$= \frac{7x}{10}$$

Difference of vote between winning & defeated candidates=15000

i.e.

$$= \frac{7x}{10} - \frac{3x}{10} = 15000$$

$$= \frac{7x - 3x}{10} = 15000$$

$$= \frac{4x}{10} = 15000$$

$$= 4x = 150000$$

$$= x = \frac{150000}{4}$$

$$= x = 37500$$

So total votes=37500

Now votes of the winning candidate

$$= \frac{7}{10}(37500)$$

$$= 26250 \text{ votes}$$

Example 6

In a college examination 52% of the candidates failed in mathematics & 42% failed in English .If 17% failed in both the subjects, then the percentage of the candidates who pass in both subject is?

Sol:

Candidates failed in mathematics =52%

Candidates failed in English =42%

Candidates failed in both subjects=17%

Candidates passes in both subjects=?

Candidates failed in at least one subject = (52%+42%)-17%

=94%-17%

=77%

Therefore candidates passed in both subjects=100%-77%=23%

Example 7

Nadeem spends 30% of his income on food articles, 40% of the remaining on conveyance & clothes & saves 50% of the remaining .If his monthly salary is 18400Rs, how much money does he saves every month?

Sol:

Income of Nadeem=Rs18400

$$Expenditure\ on\ food\ articles = \frac{30}{100}(18400)$$

$$= 5520$$

Remaining amount=18400-5520=Rs12880

40% of remaining, expenditure on conveyance & clothes=$\frac{40}{100}(12880)$

=5152

Saving is 50% of remaining

Now remaining = 12880-5152

= 7728

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So

$$= \frac{50}{100}(7728)$$

$$= \frac{1}{2}(7728)$$

$$= 3864$$

Example 8(CSS-2017)

A man buys 5Kg of meat at Rs500/Kg.In addition, for every kilogram of purchased, he has to pay a consumption tax of 6% on the selling price. Calc the total amount of money that he has to pay.

So:

Price of 1Kg meat =Rs500

Price of 5Kg meat =5×500=Rs2500

As he has to pay 6% tax on price, amount of money to be paid

$$= 2500 + \frac{6}{100}(2500)$$

$$= 2500 + (1 + \frac{6}{100})$$

$$= 2500 + (1 + 0.006)$$

$$= 2500(1.06)$$

$$= Rs2650$$

More practice problems:

Question 1: When 40% of a number is added to 42, the result is the number itself. Find the number. (70)

Question 2: A metal bar weighs 8.15 ounces. 93% of the bar is silver, how many of silver are in the bar? (7.57)

Question 3: 15 liter of a mixture contains 20% alcohol and the rest is water. If 3 liters of water be mixed in it, what is percentage of alcohol in the new mixture? $(16\frac{2}{3})$

Question 4: A student earned a grade of 80% in math that had 20 problems. How many problems in that test did the student answer correctly? (16)

Question 5: 1Kg of tea and 4Kg of sugar cost Rs35, but if sugar rises by 50% and tea 10% they would cost Rs42.50 .Find the price per Kg of sugar.

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Chapter 4: Ratio, Rates

Definition

"The comparison of quantities with same units is called ratio."

It is represented as:

a : b

Where a=antecedent

b = consequent

$$a : b \neq b : a$$

e.g.

$$1 : 2 \neq 2 : 1$$

$$\frac{1}{2} \neq \frac{2}{1}$$

$$\frac{1}{2} \neq 2$$

Example 1

Monthly income of Ali's father is 40,000/-while expenditure is 35000/-

1-Find the ratio between expenditure and saving.

2-Ratio between income and expenditure.

3-Ratio between income and saving

Sol

1-Expenditure: saving

As

Income =Expenditure + Saving

Saving = Income – Expenditure

Saving = 40,000 – 35000

Saving = 5000

Now

Expenditure : Saving

=35,000 : 5000

= 35000/5000

=7/1

= 7 : 1

2-Income : Saving

40,000 : 5000

=40000/5000

=8/1

= 8 : 1

3- Income : Expenditure

Income : Expenditure

=40,000 : 35000

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=40,000/35000

=8/7

=8: 7

Example 2

If 40% of a number is equal to two-third of another number, what is the of first number to the second?

Sol:

Let the first number be **x** and the second number be **y**

Then

$$40\%(x) = \frac{2}{3}(y)$$

$$\frac{40(x)}{100} = \frac{2(y)}{3}$$

$$\frac{4x}{10} = \frac{2y}{3}$$

$$\frac{x}{y} = \frac{2}{3} \times \frac{10}{4}$$

$$= \frac{20}{12}$$

$$\frac{x}{y} = \frac{5}{3}$$

$$x : y = 5 : 3$$

So first number is 5 and second number is 3

Example 3

Sum of the money is divided among 3 persons A, B and C in the ratio 10:7 :5.If 'B' gets Rs.140 more than 'C',how much 'A' get?

Sol:

Let shares of A ,B and C be

$$A = 10(x) \quad (1)$$

$$B = 7(x) \quad (2)$$

$$C = 5(x) \quad (3)$$

As B gets 140 rupees more than C so,

$$B = C + 140$$

$$C + 140 = 7(x)$$

$$5(x) + 140 = 7x$$

$$140 = 7x - 5x$$

$$140 = 2x$$

$$x = \frac{140}{2} = 70$$

Therefore

$$A = 10(70) = 700$$

$$B = 7(70) = 490$$

$$C = 5(70) = 350$$

So

A=700 rupees

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Example 4

If you have 24 coins and ratio of pennies to nickels is 2:1 ,how many of type of coin do you have?

Sol:

2 : 1 means 3 parts of 24 coins

No. of coins in each part = 24/3=8

As pennies are 2 so,

$$2(8) = 16$$

Nickels

$$1(8) = 8$$

Hence, 16 coins are pennies and 8 coins are nickels.

Example 5

At a camp for boys and girls, the ratio of boys to girls 3:5 .If camp enroll is 160, how many of the children are boys?

Sol

3 : 5 means 8 parts of 160.

No. of children in one part = 160/8=20

Now, three parts are boys and 5 parts are girls.

So ,

Boys = 3(20)=60

Girls = 5(20)= 100

For total again 100 + 60 = 160

Example 6 (CSS -2017)

Divide Rs.500 among Arham, Mariam and Saim so that Arham gets 2/3 of what Mariam gets and Mariam gets ¼ of what Saim gets. Find the share of each.

Sol:

Arham : 2/3 (M)

Mariam : ¼ (S)

Now

Arham : Mariam : Saim

2/3(M) : ¼(S) : 1

2/3(1/4) :1/4(1) : 1

2/12 : ¼ :1

2 : 3 : 12

so total parts =2+3+12=17 of 500

Now, share of **Arham** = 2/17(500) =58.82

Share of Mariam =3/17(500) =88.23

Share of Saim =12/17(500) =352.94

Example 7

Find the compound ratio of 3:4 , 8:15 & 25:28

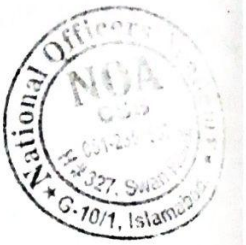
Sol

3: 4 , 8: 15 *and* 25: 28

$$= \frac{3}{4} \times \frac{8}{15} \times \frac{25}{28}$$

$$\frac{2}{5} \times \frac{25}{28}$$

$$\frac{5}{14}$$



5 :14 is the compound ratio.

Example 8

The ratio of the present ages of father and son is 8:3.After 10 years the rat their ages would be 2:1 .Find the present age of father.

Sol:

$$8(x):3(x)$$

$$8x + 10 : 3x + 10 = 2:1$$

$$\frac{8x + 10}{3x + 10} = \frac{2}{1}$$

$$8x + 10 = 6x + 20$$

$$8x - 6x = 20 - 10$$

$$2x = 10$$

$$x = 5$$

Now

Age of father is =8(5) =40

Similarly, age of son is =3(5) =15

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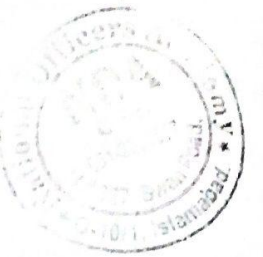
Practice problems

Question 1: In a class, the number of boys is more than the number of girls by 12% of the total strength of the class. Find the ratio of boys to girls. (14/11)

Question 2: Find the ratio of 3.5 Kg to 280 grams. (25:2)

Question 3: Two numbers are in the ratio 3:4 .If 6 is added to each term of the ratio, there is an increase of 20 percent in the given ratio. Find the first and second number. (3, 4)



Rate

"A ratio that compares two quantities of different units is called rate."

For example

Speed is a rate of change of distance

$$V = \frac{distance}{time} = \frac{s(unit\ of\ metres)}{t(unit\ of\ time)}$$

$$V = \frac{s}{t}$$

Example 1

A snail crawls about 2 feet in 1 hour. How many minutes will it take to cr inches?

Sol

$$Given\ rate = \frac{2ft}{1hr}$$

Step 1: convert the given rate into desired units first.

$$\frac{2ft}{1hr} = \frac{2(12\ inches)}{60\ min} \quad (as\ 1ft = 12\ inches)$$

$$\frac{2ft}{1hr} = \frac{24\ inches}{60\ min}$$

So,

$$\frac{24}{60}\ inches/min$$

$$original\ rate = \frac{24}{60}\ inches/\min \qquad (1)$$

$$Desired\ rate = \frac{7\ inches}{x\ \min} \qquad (2)$$

Equating the two rates for unknown 'x'

$$\frac{24}{60} = \frac{7}{x}$$

$$24x = 7 \times 60$$

$$24x = 420$$

$$x = \frac{420}{24}$$

$$x = 17.5$$

Example 2

A clerk walks from his house at 4km/h and reaches his office 5 minutes late. If his speed is 5Km/h, he will reach his office 10 minutes early. How far is his office from home?

Sol:

Let the distance he has to travel be= dKm

Time taken to travel this distance at 4Km/hr=$\frac{d}{4}$hrs

Time taken at $\frac{5Km}{hr} = \frac{d}{5}\ hrs$

Difference in time = $\frac{d}{4} - \frac{d}{5} = \frac{d}{20}$

Actual Difference = 15 min = $\frac{1}{4}\ hrs$

So,

$$\frac{d}{20} = \frac{1}{4}$$

$$4d = 20$$

$$d = \frac{20}{4} = 5$$

$$d = 5Km$$

Practice problems

Question 1: A train running between two stations arrives at its destination late when it travels at 40Km/hr. and 16 min late when it travels at 30 Km/hr the distance between two stations. (12 Km)

Question 2: Arshad can cover a circular path of radius 21m in 44 seconds much time he will take to cover a distance of 3Km? (16 min and 40 sec).

Question 3: What is the speed of 75Km long train which passes a 150m platform in 10 seconds? (81Km/hr.)

Chapter 5: Proportion

Definition

"Equality between two ratios is called proportion".

$$a : b = c : d$$

$$a : b :: c : d$$

Types:

Two types or proportions are commonly used:

1) Direct proportion
2) Inverse proportion

Direct proportion:

When one quantity is increased the other also increases and on decreasing one quantity other also goes on decreasing, so there is direct proportion between two quantities.

For instance, when you increase force on the accelerator of a car while driving its speed also increases, so there is a direct relation/proportion between force and speed.

Inverse proportion:

When one quantity is increased other is decreased and vice versa then there is an inverse proportion between two quantities.

For instance, when load is increased on a vehicle its speed is decreased and vice versa; so, there is inverse proportion between load and speed.

Example 1:

20 pens cost 200 rupees, what will be the cost of 35 pens? (Simple proportion with two things).

Sol:

By definition, ratio is comparison of two or more things with same units.

So,

$$20 : 35 :: 200 : x \text{ (pens with pens and rupees with rupees)}$$

$$\frac{20}{35} = \frac{200}{x}$$

$$20x = 35 \times 200$$

$$x = \frac{35 \times 200}{20}$$

$$x = \text{Rs. } 350$$

Other method

By Formula:

Product of means=product of extremes

Means

$20 : 200 :: 35 : x$

Extremes (Corresponding values in this method)

$$20(x) = 35(200)$$

$$x = \frac{35(200)}{20}$$

$$x = 350 \text{ Rupees}$$

Example 2:

The price of 80 shirts is 22000 rupees. What will be price of 30 shirts?

Sol

$$80 : 30 : \quad 22000 : x$$

$$\frac{80}{30} = \frac{22000}{x}$$

$$80x = 22000 \times 30$$

$$x = \frac{22000 \times 30}{80}$$

$$x = 8250 \; rupees$$

Compound proportion (with more than two things)

Use arrow method to solve such type of proportion.

Arrow method includes 3 steps:

1- Construct a table i.e. rows and columns.
2- Draw arrows; 1stly in column with unknown variables then comparing all the other columns individually with unknown column keeping the other known column constant.
3- Solve the proportion; keeping the unknown ratio (follow direction of arrows) on one side of equal and all other ratios multiplied with each other on other side of equality.

N.P (Notable point)

In case of simple proportion, also use arrow if there is an inverse proportion between the two quantities. Then, following the direction of arrows, it may be solved by definition of ratio. These examples will make it clear:

Example 3:

6 men work for 12 days to finish a task if more two men are added, how days will it take to finish the same task?

Men	Days
6	12
8	X

Now by definition

$$x : 12 :: 6 : 8$$

$$\frac{x}{12} = \frac{6}{8}$$

$$x = \frac{6}{8} \times 12$$

$$x = 9\ days$$

Example 4:

An army camp has a 30 day stock of food for 900 soldiers. 150 soldiers lea camp. Tell, for how many days same stock of food will be sufficient for soldiers?

Sol:

Soldiers	Days
900	30
750	X

$$x : 30 :: 900 : 750$$

$$\frac{x}{30} = \frac{900}{750}$$

$$x = \frac{900}{750} \times 30$$

$$x = 36\ days$$

Example 5:

A fort had enough food for 80 soldiers for 60 days. How long would the food last if 20 more soldiers join after 15 days?

Sol

Days		Soldiers	
45= 60 - 15	↑	80	↓
X		100	

$$\frac{x}{45} = \frac{80}{100}$$

$$x = \frac{80}{100} \times 45$$

$$x = 36$$

Now for 15 days later

$$x = 51\ days$$

Example 6:

Five men work 8 hours daily to manufacture 200 commodities .If 8 men w 6 hours daily then how many commodities will be manufactured?

Sol

Men	Working Hours	Commodities
5	8	200
8	6	X

$$\frac{x}{200} = \frac{6}{8} \times \frac{8}{5}$$

$$x = \frac{6}{5} \times 200$$

x = 240 commodities

Example 7:

If 4000 rupees are sufficient for a family of 4 members for 40 days then fo many days 15000 rupees will be sufficient for a family of 5 members?

Sol

Members of family	Days	Rupees
4	40	4000

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5	X	15000

$$\frac{x}{40} = \frac{4}{5} \times \frac{15000}{4000}$$

$$x = 120 days$$

Practice problems

Question 1: 14 cows eat 63Kg grass in 18 days. How many cows will eat 770Kg grass in 28 days? (110)

Question 2:A food factory manufactures 560 fans in 7 days with 20 machines. How many fans would be manufactures in 12 days with 18 machines?

Question 3: The cost of 16 packets of salt, each weighting 900 grams is 84 dollars .what will be the cost of 27 packets of salt each weighting 1 Kg? (157.50)

Question 4: If 270 Kg of corn would feed 42 horses for 21 days, for how many days would 360 Kg of it feed 21 horses? (56)

5	X	15000

$$\frac{x}{40} = \frac{4}{5} \times \frac{15000}{4000}$$

$$x = 120 days$$

Practice problems

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Assignment: General science & abilities

Question 1: 14 cows eat 63 Kg grass in 18 days. How many cows will eat 770Kg grass in 28 days?

Question 2: A factory manufactures 560 fans in 7 days with 20 machines. How many fans would be manufactured in 12 days with 18 machines?

Question 3: The price of 80 shirts is Rs.22000. What will be price of 30 shirts?

Question 4: a) Hamza spends 20% of his total income on house rent, 70% on domestic expenditure. If his savings is Rs.1800. What will be his total income?

b) Change into fractions 70%

c) Find 15% of 600.

Question 4: a) which fraction is larger in the following?

$$\frac{7}{9}, \frac{1}{4}, \frac{13}{36}$$

b) Solve:

a) $(7)^2 + x - (2 \times 4) \div 2$
b) $9 + 3 + 3 \times 2$
c) $(x^2)^3 = ?$
d) $x^a . x^b = ?$, $\frac{x^{a+b}}{x^{c-d}} = ?$
e) ***convet into meter*: 10 *cm***

Chapter 6: Average/Mean, Median, Mode & Range

Average:

Why do we take average?

It reduce the error in calculations and leads to accuracy.

$$Average = \frac{Sum\ of\ observation}{No.\ of\ observation}$$

Example 1:

1) Find the average of 4, 7 & 10:

Sol

$$Average = \frac{4+7+10}{3}$$

$$= \frac{21}{3}$$

$$= 7$$

2) Find the average of 1, 3, 5, 6, 7, 8

Sol

$$Average = \frac{1+3+5+6+7+8}{6}$$

$$= \frac{30}{6}$$

$$= 5$$

Example 2:

The average marks of 3 batches of 55,60 and 45 students respectively is 50,55 and 60.Then the average marks of all students =?

Sol:

We know that

$$Average = \frac{Sum}{Number\ of\ observatios} \qquad (1)$$

From (1) "sum" can also be expressed as:

$$Sum = Average \times Number\ of\ students$$

Now

$$Average\ of\ all\ students = \frac{Sum\ of\ students}{No.\ of\ students}$$

$$= \frac{Average \times No.\ of\ students}{No.\ of\ students}$$

$$= \frac{(50 \times 55) + (60 \times 55) + (45 \times 60)}{45 + 55 + 60}$$

$$= \frac{2750 + 3300 + 2700}{160}$$

$$Average = \frac{8750}{160} = 54.68$$

Example 3:

The average weight of A, B and C is 45Kg (weight has unit of Newton but due to constant force of gravity of Earth i.e. 9.8m/s^2 it is sometimes expressed in units of mass i.e. Kg).If the average weight of A &B is 40Kg and that of B&C is 43Kg then find the weight of B.

Sol:

$$Average = \frac{Sum}{No.\ of\ observations}$$

$$45 = \frac{A+B+C}{3}$$

$$A + B + C = 135 \qquad (1)$$

$$40 = \frac{A+B}{2}$$

$$A + B = 80 \qquad (2)$$

Also

$$\frac{B+C}{2} = 43$$

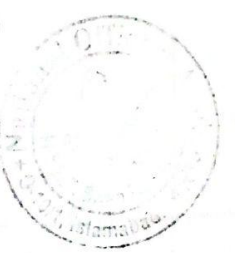
$$B + C = 86 \qquad (3)$$

Adding (2) & (3)

$$A + B + B + C = 80 + 86$$

$$A + 2B + C = 166 \qquad (4)$$

Now for B. subtracting (1) from (4)



$A+2B+C=166$

$-A\pm B\pm C=-135$

$B=31$

So. **B=31Kg**

Assignment

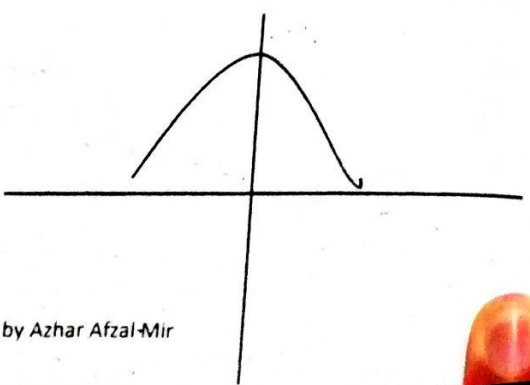
The average monthly income of 'P' and 'Q' is Rs.5050.The average monthly income of 'Q' an 'R'is Rs.6250 and average monthly income of P and R is Rs.5200.Find the average monthly income of P.

Median

"The measure of central tendency in the data is called median."

- Write data in ascending order first.

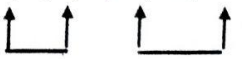
For example, well known graph of Gaussian distribution:



The value representing the center of graph would be termed as **median,** physically

For odd values in data

Find median of 1, 4, 7, 9,12, 14, 15



Median = 9

For even values in data

Find median of 1, 4 ,7 , 8 , 10, 12 ,16 ,20

$$Median = \frac{8+10}{2}$$

$$Median = \frac{18}{2}$$

$$Median = 9$$

Mode:

"The statistical term used to refer the frequently occurring value in the data"

Or

"Most repeated value in data".

e.g. 1,4, 5 , 6 , 6 , 7

Mode= 6

1 , 4 , 5 , 6 , 6 , 7 , 8 , 9 , 9 , 9 ,10

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Mode = 9 (most repeated)

1 , 4 , 5 , 5 , 6 , 6, 7, 8

Mode =5 , 6

Range:

"The difference between maximum and minimum value of data."

e.g. 2,4,5,6,7, 9, 12

Range = 12-2 = 10

Practice problems

Question 1: There are 9 students in a group having ages 15 , 15 , 16 , 16 , 16 ,17 , 17 , 18 , 19.calculate mean, median, mode and range also define above mathematical terms. (CSS – 2016)

Question 2: The front row of the movie theatre has 23 seats. If you were asked to sit in the seat that occupied the median position, on which seat you have to sit?

Question 3: A student recorded her scores on weekly math quizzes that were marked out of a possible 10 points. Her score were as follows:

8 , 5 , 8 , 5 , 7 , 6 ,7 , 7 ,5 ,7 ,5 ,5 ,6 , 6 , 9, 8 , 9 , 7 , 9 , 9 ,6 , 8 , 6 , 6 ,7 what is the mode for her score on the weekly math quizzes?

Question 4: Rashid buys three books for Rs.16.00 each and four books for Rs.23.00 each, what will be the average price of books?

Question 5: A student gets 75, 82, 86 marks on three tests. What must be his mark on the next test, so that his average for the four tests will be 85? (97)

Question 6: A car travelled 100Km with half the distance at 40Km/h and the other half at 80Km /h. Find the average speed of the car for the whole journey. 53.33km/h)

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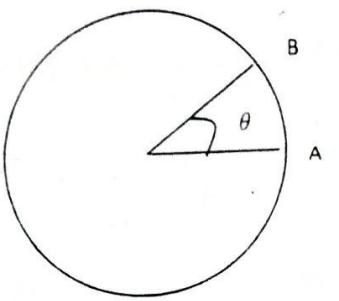
Chapter 7: Basic Geometrical Concepts

(Angles, Triangles, Geometrical shapes, Areas, Volume)

Angle:

"Angle specifies the angular displacement of a particle or body moving along the circular path."

Let we consider a body moving on the circular path e.g. you are moving on a merry-go-round, cover some angular displacement moving from point A to B which is specified by angle.



Angle is represented by a Greek letter "θ" and by "$\angle$" symbolically

Units:

An angle shows a physical quantity which can be measured by some units easily; so there are three units commonly used for angular displacement:

1) Degree
2) Revolution
3) Radian

Degree is commonly used unit (you have used frequently in matric for solving trigonometric questions, preferred for smaller systems.

Revolution is used for larger bodies and larger circular orbits e.g. The Earth revolves around the sun, its angular displacement will be expressed in revolution.

Radian is system international unit of angular displacement used in the whole world commonly.

Types:

1) **Right angle:** Angle of 90 degree.
2) **Acute angle:** Angle less than 90 degree.
3) **Obtuse angle:** Angle greater than 90 degree and less than 18 degree.

Triangles

Tri ---------- Three

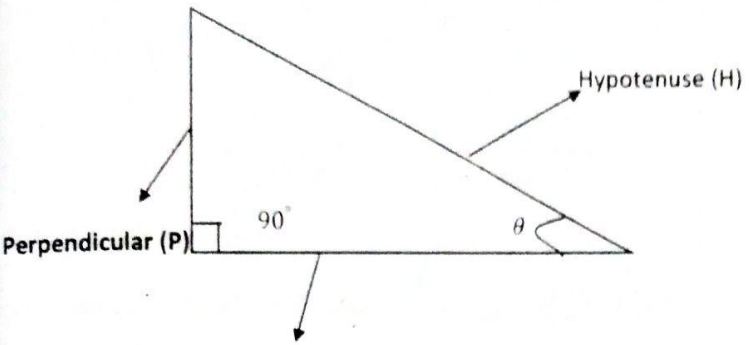
"A closed figure with 3 angles and three 3 sides is called triangle."

Types:

- Right angle triangle
- Oblique triangle
- Equilateral triangle
- Isosceles triangle
- Scalene triangle
- obtuse angle triangle
- Acute angle triangle

Right angle triangle:

"A triangle with an angle of 90 degree is called right angle triangle."



Some important trigonometric ratios for the solution of right angle triangle:

$$\sin \emptyset = \frac{Perpendicular}{Hypotenus}$$

$$\cos \emptyset = \frac{Base}{Hypotenus}$$

$$\tan \emptyset = \frac{Perpendicular}{Base}$$

Pythagorean Theorem

It states that

"Square of hypotenuse of right angle triangle is equal to sum of squares of two other side."

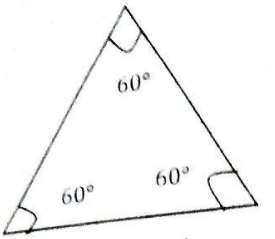
$$H^2 = B^2 + P^2 (1)$$

Oblique triangle:

"The triangles that are not right angle triangles are called oblique triangles.

Equilateral triangle:

"A triangle with all three sides and three angles equal."

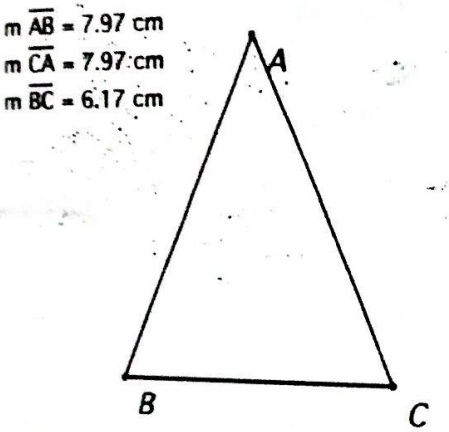


As sum of angles of a triangle = 180°

i.e. (60 + 60 +60 = 180) in this case

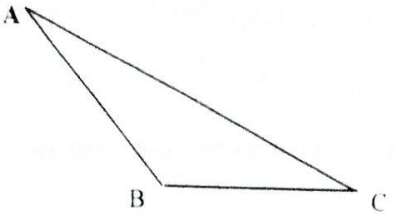
Isosceles triangle:

"A triangle with two sides and two angles equal is called isosceles triangle."



Scalene triangle:

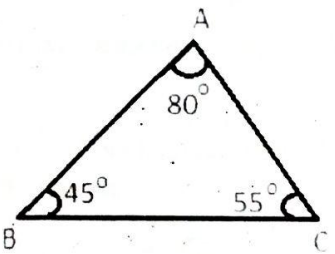
"A triangle with all sides and all angles different is called scalene triangle."



Acute angle triangle:

"A triangle with all three angles less than 90° is called acute angle triangle."

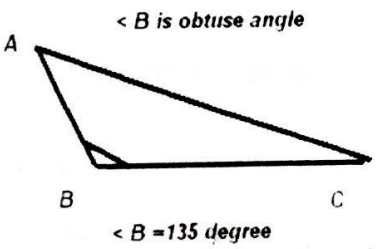
e.g.



Acute angle triangle (all angles < 90°)

Obtuse angle triangle:

"A triangle with one obtuse angle (> 90°) is called obtuse angle triangle."



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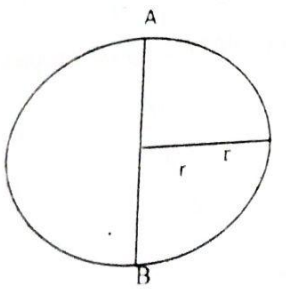
Some geometrical shapes with area and volume

1) Circle
2) Sphere
3) Rectangle
4) Trapezium
5) Square
6) Triangle
7) Rhombus
8) Cylinder

1) Circle

"A round figure whose boundary consists of points equidistant from a fixed point (center), called circle."



$$Diameter = AB$$

A straight line passing through center connects two points on the boundary of circle, called circumference.

$$Radius = half\ of\ diameter$$

$$R = \frac{d}{2}$$

$Area\ of\ circle = \pi r^2$ (Where r= Radius)

$Circumference(boundary) = 2\pi r$ (Where r= Radius

Angle of circle = 360°

(*It comes from natural revolution of the Earth around the sun*)

2) Sphere

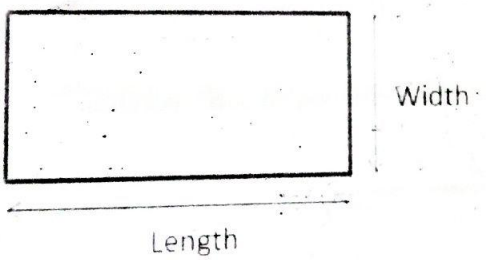
"A three dimensional circle e.g. Football, Earth, and ball."

$Volume\ of\ sphere = \frac{4}{3}\pi r^3$ (Where r = Radius of sphere)

$Area = 4\pi r^2$ (Where r = Radius of sphere)

3) Rectangle

"A close figure with four unequal adjacent sides and four right angles."

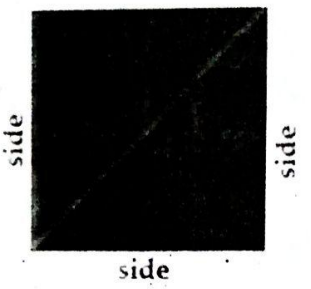


Area of rectangle $= Length\ X\ Width$

4) Square

"A closed geometrical figure with four equal sides and four right angles."

Square & Formulas



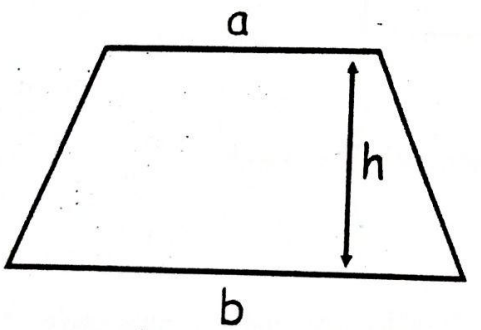
$$Area = side * side$$

$$Perimeter = 4 * side$$

$$Diagonal = \sqrt{2} * side$$

5) Trapezium

"A four-sided figure with one pair of sides parallel."



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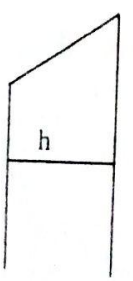

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$$Area = \frac{1}{2}[(sum\ of\ parallel\ sides) \times (perpendicular\ distance\ b/w\ parallel\ sides)]$$

6) Triangle

"A closed geometrical figure with three angles and three sides."

Normally, we define for right angle triangles as:

$$Area = \frac{1}{2}(Base \times Altitude)$$

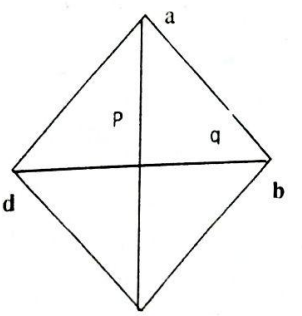
7) Rhombus

"A four-sided figure with same length."

$$Area = \frac{1}{2}(product\ of\ diagnols)$$

$$= \frac{1}{2}(p.q)$$

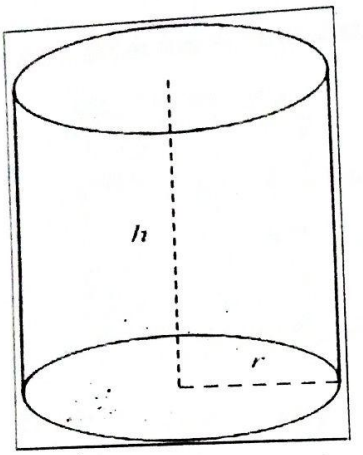
$$where\ p = ac\ and\ q = bd\ (in\ below\ figure)$$



c

8) Cylinder

"A three-dimensional surfacebounded by a curved surface and two parallel circles of equal size at the ends."



$$Area = \pi r^2 + \pi r^2 + 2\pi rh$$

$$Area = 2\pi r^2 + 2\pi rh$$

And

$$Volume = \pi r^2 h$$

Practice problems:

Question1: A window has a length of 8 feet and width of 6 feet. What is the area?

Question 2: A triangular –shaped yard has a base of 25 meters and a height of 12 meters. What is its area?

Question 3: The perimeter of a square is 188cm. What is the length of each side?

Question 4: A triangular-shaped rug has a base of 7 meter and a height of 6 meter .What is its area?

Question 5: Find the volume of the brick with length of 24cm, width 12cm and height 6cm respectively.

Question 6: Find the perimeter of hexagon with a side length of 10 meter.

Question 7: A square has an area of $225 cm^2$.what is its perimeter?

Question 8: Find the area of circle which has diameter of 8cm.

Question 9: Find the volume of a football with radius of 10cm.

Question 10: A Triangle contains three angles, if two angles are 45 degree and 90 degree respectively, Find the 3rd angle of the triangle.

Chapter 8: Set Theory

"A collection of distinct objects or numbers is called set."

Each number of the set is called element of set. For instance,

Set of first 5 natural numbers = {1, 2, 3, 4, 5}

Elements

Types: Following are some common types:

Empty set: A set with no element, called empty/void/null set.

Represented by { } or ∅

Singleton set: A set with one element, called singleton set.

Sub-Set: A set 'A' is the sub-set of set 'B' if all the elements of set 'A' are prese in set 'B'.

e.g. A={1 , 2 , 3 , 4} , B={ 1 , 2 , 3 , 4 ,7}

So set A is sub-set of B

Proper sub-set:

If 'B' is a proper sub-set of 'A' then all elements of 'B' are in 'A' but 'A 'contain at least one element that is not in 'B'.

Finite set: A set with finite elements

e.g. {set of days of week}

Infinite set: A set with infinite elements

e.g. {set of all real numbers}

Union of sets: Set of all elements belonging to set 'A', set 'B' or non-duplicate elements of A & B both.

e.g. A={ 1 , 2 , 3 , 4} B={ 1 ,5 ,6}

A∪B= {1, 2, 3, 4, 5, 6}

Intersection of set: Common elements of sets is called intersection

A= {1, 2, 3, 4} B= {2, 3, 5, 6}

A∩B= {2, 3}

Universal set: Set containing all objects/elements and of which all other given sets are subsets.

Complement of set 'A': Difference of universal set and set 'A', called complement of set 'A'. i.e. $A^c/A' = U - A$

Difference of two sets: Difference of two set, expressed as A-B is the set of all elements of 'A' that are not element of 'B'. e.g.

A= {1, 2, 3, 4, 5} , B= {2, 4, 5}

A-B= {1, 3}

De-Morgan's Laws in set theory:

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Example: prove $(A \cap B)' = A' \cup B'$ for given sets

U= {a, b, c, d, e, f}

A= {b, d, f}

B= {f, b, c}

L.H.S =

A∩ B= {b, d, f} ∩ {f, b, c}

A∩ B= {b, f}

Now $(A \cap B)' = U - (A \cap B)$

= {a, b, c, d, e, f} – {b, f}

$(A \cap B)' = \{a, c, d, e\}$ (1)

R.H.S =

$A' \cup B'$ =?

$A' = U - A$

= {a, b, c, d, e, f} – {b, d, f}

A'= {a, c, e}

B' = U – B = {a, b, c, d, e, f} – {f, b, c} = {a, d, e}

Now

$$A' \cup B' = \{a, c, e\} \cup \{a, d, e\}$$

$$A' \cup B' = \{a, c, d, e\} \quad (2)$$

From (1) & (2), proved

$$(A \cap B)' = A' \cup B'$$

Similarly you can prove other i.e.

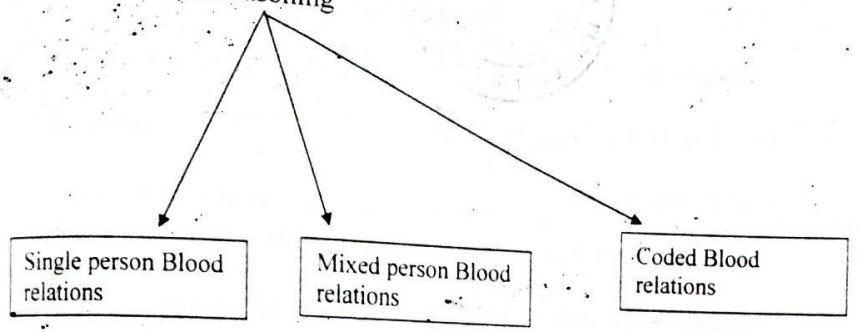
$$(A \cup B)' = A' \cap B'$$

Chapter 9: Logical problems

Derived from Greek word **"Logos"** meaning **"Reasons"**.

"The problems dealing with correct reasoning for a conclusion, are called logical problems."

- Alphabetic series test.
- Alphanumericseries test.
- Coding – Decoding
- Blood relation reasoning



- **Alphabetic series test.**

1) Write 26 alphabets on page in order to read quickly letters in forward direction or in backward direction.
2) You must be familiar with one-step forward, two-step forward and so on, similarly for one-step backward, two-step-backward etc. One step forward means jumping on next consecutive letter and one step backward means jumping on consecutive previous letter.
3) Handle one letter or number at a time.

Example 1:

AZ, CX, FU, **JQ**

Example 2:

DKY, FJW, HIU, JHS, **LGQ**

Alphabets

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z

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- **Alphanumeric series test.**

In such that alphabet and numbers are mixed up. Use alphabets technique in t test also for letters

Example 1:

Q1F, S2E, U6D, W21C, **Y88B**

Example 2:

D-4, F-6, H-8, J-10, **L-12**, **N-14**

- **Coding De-coding.**

Example 1:

If **friend** is coded as HUMJTK, how **candle** be written in that code?

Sol: candle = EDRIRL

Example 2:

If **"PALE"** is coded as2134,**"EARTH"** is coded as 41590, how c **"PEARL"** be coded in that language?

Sol: PEARL = 24153

- **Blood relation reasoning**

Single blood relations

Such problems involve the relation of one person to another individual e.g.

Example 1:

John introduces Marry as the daughter of only son of my father's wife. How Marry related to John?

Sol:

Teqniques to solve person blood relations:

1) Break the sentence from the introductory words like as/was/is etc.
2) You must be familiar with the blood relations like brother's daughter is "Niece "sister's son is "Nephew" and such other relations on paternal and maternal sides.
3) Start writing relations in simple terms from end of introductory sentence.e.g. "The Daughter of only son of my father's wife".

$$Johan's\ Father's\ wife = Johan's\ mother$$

$$Only\ son\ of\ mother = John\ himself$$

$$Daughter\ of\ John's\ himself$$

So Marry is daughter of John .

Example 2:

Pointing to a lady Ravi said, she is the only daughter of the father of my sister's brother, what is relation of Ravi and Lady?

$$My(Ravi)\ sister's\ brother = Brother$$

$$Only\ daughter\ of\ Ravi's\ father = Ravi'sister$$

So, Ravi and lady are brother sister.

Example 3: (Assignment)

Introducing Saad, Amina said, his brother's father is the only son of my grandfather. How Amina is related to Saad?

Example 4: (Assignment)

Pointing to a man in photograph a woman says that his mother's only daughter is my mother. How is man related to woman?

Example 5: (Assignment)

A told B that C is his father's nephew. B is A's cousin, but not the brother of C. What relationship is there between D & C? (CSS-2017)

Mixed blood relation:

Such problems contain more than one relations i.e. multiple individuals are mixed up. You are to find a specific relation among all that relations.

Example 5

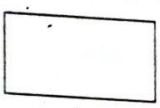
X, Y are brother-sister C & D are wife and husband. X is the son of 'C' and 'D is sister of 'D. How 'Y' is related to 'F'?

Sol:

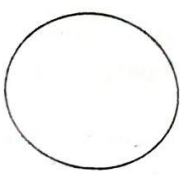
Teqniques to solve the mixed blood relations:

1) Use family tree symbols as follows:

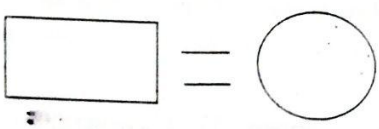
Male:



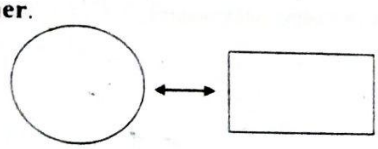
Female:



Husband-Wife



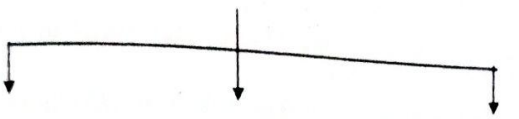
Sister- Brother.



1) **Child:**

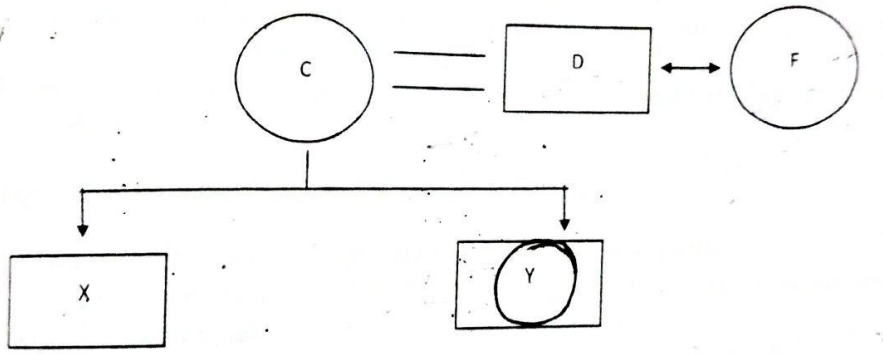
2) **Children**

3) **Children**



And so on

Now



So, **'Y'** is Niece of **'F'**

Coded-Blood relations

Example 6

If P+Q means 'P'is husband of 'Q' , P/Q means 'P' is sister of 'Q' , P*Q means 'P' is son of 'Q'. How 'D' is related to 'A' in expression:

D*B+C/A

Sol:

C/A ⟶ C is sister of 'A'

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B+C ⟶ B is husband of C

'B' is brother-in-law to 'A'

D*B ⟶ 'D' is son of 'B'

So, D is Nephew to 'A'

Chapter 10: Analytical problems

Sources:

"The thinker's Tool Kit"

By Morgan Dons (Former CIA analyst)

G.R.E (general)

"Analytical" derived from "analysis"

Means "separating a problem into its constituent elements."

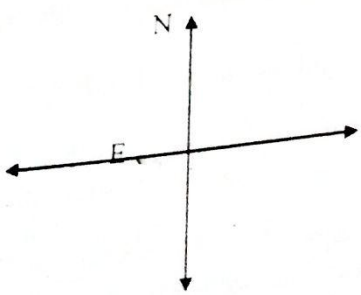
It reduce the complex issues to their simplest terms.

Introduction to analytical problems:

- Each analytical problem begins with a few introductory sentences describing a particular situation.
- The introduction will tell you how many people or things you are going to be working within the problem. (Typically asks to manipulate between 5 & 8 people/thing, sometimes more, some times less)
- The introduction is followed by set of certain conditions that must be followed:

Strategy to tackle analytical problems:

- These problems require great attention to few minutes' details.
- Try to diagram the information that you are given before you try to answer any of the questions.
- You must be aware of the directions like East, West, North, and South. Usually "East" is drawn on the paper with respect to your right hand when you are writing on the page "West" on the side of your left hand, "North" upside and "South" downwards vertically e.g.



***(Don't try to mix it with actual sunrise and sunset)**

For the directions of object in the problem which is to be moved follow the directions according to the right & left hand of the individual walking along particular directions. Your hands are for general (E, W, N, S) directions.

- Work in sequence and draw diagrams of all given conditions in the analytical problem (like condition A&B sit together can be represented by **A<>B**)

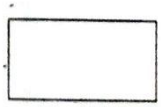
  e.g. (Some people prefer to answer first those questions about a passage that seem difficult, it is not a good practice to start another passage before finishing one. Begun earlier because it is time consuming to return to the passage and the re-establish familiarity with its relationships).

- Avoid "**Analytical sins**"e.g.

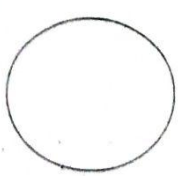
1) Don't be exercise bike in thinking and discussion i.e. spending lot of energy, sweating but goes to nowhere.
2) Don't follow instinctive approach (**instinct** - inborn pattern of activity, not learned-we prefer the solution naturally we like rather to see structural approach. In instinctive approach mind generally remains closed to alternatives. On the other hand, in structured approach, mind generally remains open to alternatives enabling one to examine each element of the decision or problem separately).

- For blood relation questions use the following symbols to solve quickly:

Male:

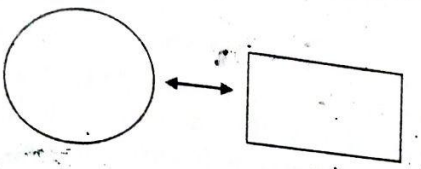


Female:



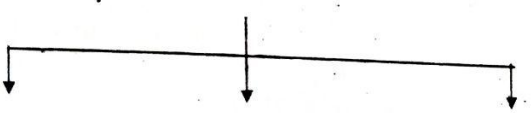
Husband-Wife

Sister- Brother.



Child:

Children



- Try to reduce the problem from 3-dimentional to 2-dimentional because you have to work on the paper and paper is two dimensional .For instance, any house in the given problem and general directions(East, West, North, South) are perceived to be 3-dimentional; so, you have to reduce in 2-dimentions.
- When circular arrangements of the people are asked in any analytical problem, only two arrangements are possible:

  People facing inside the circle and facing outside the circle.

  When people are facing inside the circle than right mean anti=clockwise and left means clockwise. It is vice versa in case of people facing outside.

- Basic arithmetic's, probability, percentages and basic geometrical techniques will help a lot regarding the analytical problems.

Following solved problems will make these instructions more clear. For maximum practice of mental, logical and analytical problems are attached at the end of the book.

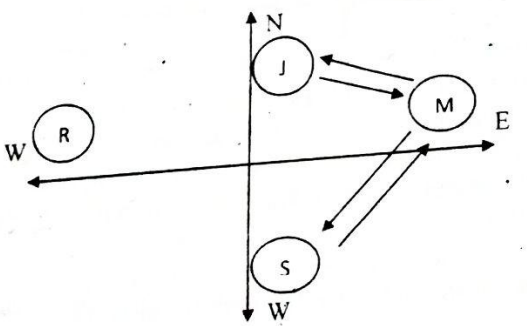
[Some common analytical problems and general strategies to tackle]

Example 1:

A four person crew from classic colors is painting Mr. Fields house. Michal is painting the front of the house. Ross is alley behind the house painting the back. Jed is painting the window frames on North side, Shawan is on South. If Michal switches places with Jed and Jed then switches places with Shawan.

1) **Where is final position of Shawan?**
2) **Where is final position of Michal?**

Sol:



Let the front portion of the house be on the East.

Drawing the introduction and conditions using arrows, it is the clear from mapping

Shawan is on front i.e. **East.**

Michal is on **North.**

Example 2:

Amrit, Bector, Chinky, Deepinder, Eeshwar, Fancy, Gurkamal and Hero are sitting around a circle facing the center. Fancy is 3rdto the right of Chinky and 2nd to the left of Hero. Deepinder is not an immediate neighbor of Chinky or Hero. Eeshwar is to the immediate right of Amrit, who is 2nd to the right of Gurkamal?

1) **Who is 2nd to the left of Chinky?**
2) **Who is to the immediate right of Chinky?**
3) **Who sits between Gurkamal and Deepinder?**

General trick for such circular arrangements:

Two types of circular arrangements of people are possible:

1) Facing inside the circle:

In this case, remember:

$$Right\ means = anti - clockwise$$

$$Left\ means = clockwise$$

2) Facing outside the circle:

$$Right\ means = clockwise$$

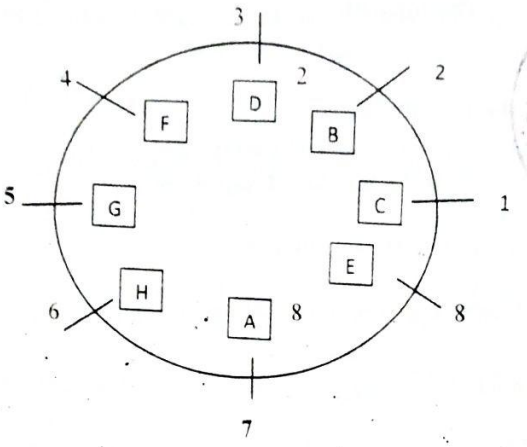
$$Left\ means = aniti - clockwise$$

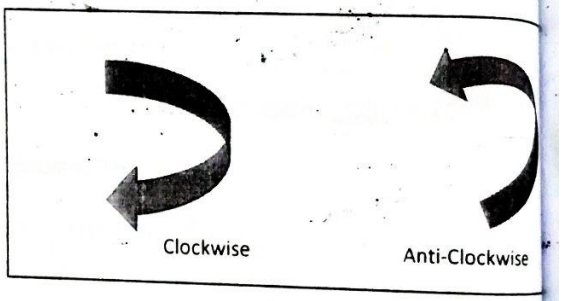
Sol:

Our case is circle with face inside; so,

$$Right = anti - clockwise$$

$$Left = clockwise$$





From diagram it's clear that

1) **Armit**
2) **Bector**
3) **Fancy**

Solve this question for facing outside

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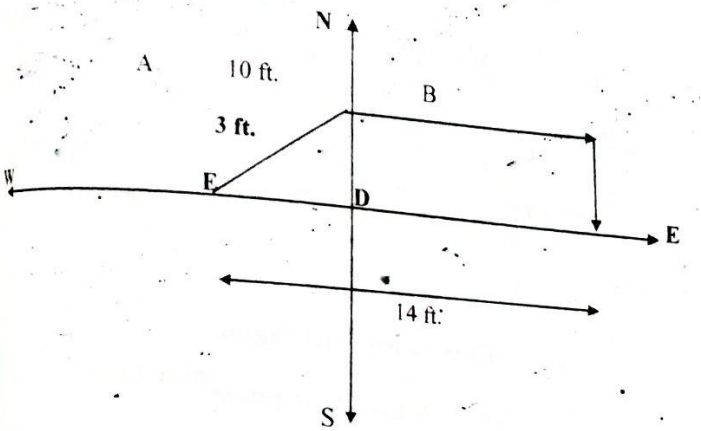
Example 3:

Reena walked from A to B in the East 10 feet. The she turned to the right and walked 3 feet. Again she turned to the right and walked 14 feet. How far is she from A?

Sol:

During general directions:



From diagram | AE| =?

Using basic geometry and applying Pythagorean Theorem:

$$(\mathbf{Hyp})^2 = (\mathbf{Base})^2 + (\mathbf{perpendicular})^2$$

$$(\mathbf{AE})^2 = (\mathbf{ED})^2 + (\mathbf{DA})^2$$

$$\mathbf{AE} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

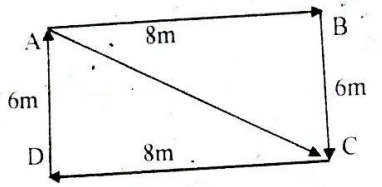
$$|AE| = 5\ feet$$

So, she is 5 feet far from point A.

Example 4:

A room has 8 meter and 6 meter length and breadth respectively. A cat runs along all four walls and then along the diagonal to catch a rat. Find the total distance covered by the cat.

Sol:



(Room)

Here,

$$AB = Length\ of\ room$$

$$BC = Breadth\ of\ room$$

$$AC = Diagonal\ of\ the\ room$$

$$Total\ distance = AB + BC + CD + DA + AC \qquad (1)$$

For AC

Using Pythagorean Theorem

$$AC = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

AC=10 m. using in (1)

Total distance = 8+6+8+6+10

Total distance = 38 meters

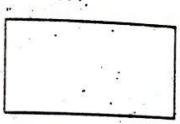
Example 5:

A's son B is married with 'C' whose sister 'D' is married to 'E', the brother of 'B'. How 'D' is related to 'A'?

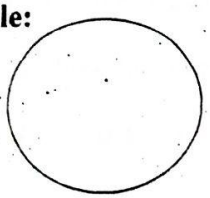
Sol:

Trick: use the following symbols to find out relations quickly, in such type of problems:

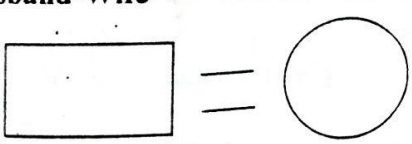
Male:



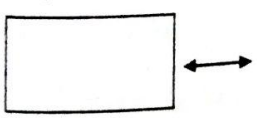
Female:



Husband-Wife

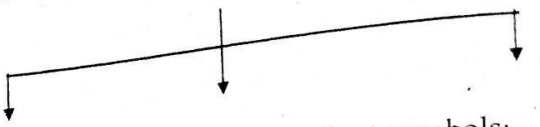


Brother-sister

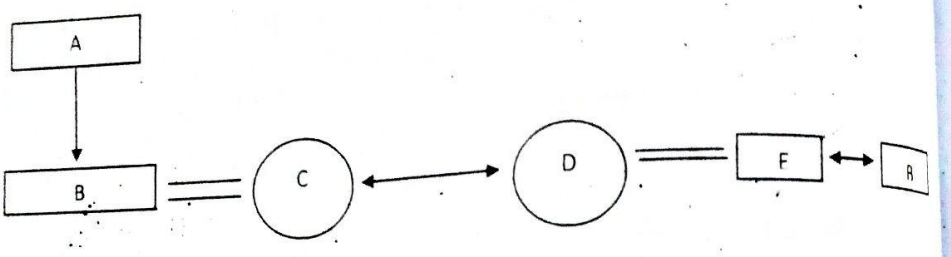


Child:

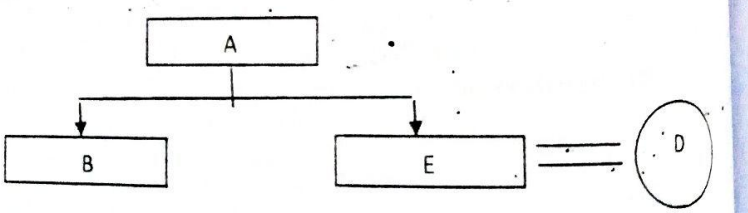
Children



Now solving the above question using these symbols:



This means



So, D is A's daughter-in-law.

Group type:

Example 6:

Ahmad, Ali, Akbar, Nasir and Shehbaz are the students of a college having different heights and weight. Ahmad weights twice as much as Ali and Ali weights five times as much as Akbar. Akbar weights half as much as Nasir and Nasir weights half as Shehbaz.

1) **Who is the heaviest in weight?**
2) **Who is the lightest in weight?**
3) **Shehbaz is lighter in weight than which of the two students?**
4) **Shehbaz is heavier than which of the two students?**
5) **Show the descending order of the weights of students.**

Sol:

In such type of height or weight comparison questions, let the last one of the chain of introduction equal to some numerical value then all others will be simple to develop in equations.

For instance, in this question Shehbaz is the last boy.

$$\textit{Let weight of shehbaz} = 60Kg \quad (1)$$

$$\textit{Nasir weight half of shehbaz} = \frac{1}{2}(60)$$

$$\textit{So,} \quad \textit{Nasir} = 30Kg \quad (2)$$

$$\textit{Akbar weights half as Nasir} = \frac{1}{2}(30)$$

$$\textit{so,} \quad \textit{Akbar} = 15Kg \quad (3)$$

$$\textit{Ali weights } 5 - \textit{times as Akbar} = 5(15)$$

$$\textit{Ali} = 75Kg \quad (4)$$

$$\textit{Ahmad weights thrice as much as Ali} = 3(75)$$

(5)

$$Ahmad = 225Kg$$

From above equations

1) Ahmad is heaviest
2) Akbar is lightest
3) Ali, Ahmad
4) Nasir, Akbar
5) Ahmad, Ali, Shehbaz, Nasir, Akbar

Example 7:

Six candidates Ali, Amin, Omef, Hamza, Saleem and Osama are being interviewed for a post. The interview will take place over four consecutive days starting on Thursday. Each candidate will have one interview. The day on which the different candidates will interview must confirm the following conditions:

1) **At least one interview will take place each day.**
2) **No more than two interviews will take place on any day.**
3) **No more than three interviews will take place on any two consecutive days.**
4) **Ali's interview must take place on Saturday.**
5) **Amin's interview must take place on the same day with another interview.**
6) **Saleem's interview must take place on a day before Osama's interview.**
7) **Osama's interview must take place on a day after Hamza's day.**

Question 1:

If only one interview take place on Thursday which candidate could have that interview?

Sol:

Thursday	Friday	Saturday	Sunday
Saleem		Ali	

Since

Saleem ⟶ Osama

Hamza ⟶ Omer

Question 2:

If the direction decides to take two interviews on Thursday and two on Sunday. How many candidates would be eligible to interviewed on Friday?

a) 1 **b) 2** **c) 3** **d) 4** **e) 5**

Sol:

Thursday	Friday	Saturday	Sunday
Hamza, Saleem	Osama	Ali	Amin, Omer
Hamza, Saleem	Omer	Ali	Amin, Osama
Amin, Hamza	Saleem	Ali	Omer, Osama
Amin, Saleem	Hamza	Ali	Omer, Osama

So 4 candidates on Friday

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Hints:

Pairs:

Amin, Omer

Amin, Osama

Amin, Hamza

Amin, Saleem

Omer, Osama

Omer, Hamza

Osama, Hamza

Omer, Saleem

Osama, Saleem

Hamza, Saleem

***6 persons**

Ali, Amin, Omer, Osama, Hamza, Saleem

*Saleem → Osama

*Hamza → Omer

*Ali = Saturday

*No more than 2 interviews on a day.

*No more than 3 interviews on consecutive days

Question 3:

If Hamza, Osama have their interviews on the same day which of the follow must be true?

A) Hamza's interview will take place on Thursday.
B) Saleem's interview will take place on Friday
C) Amin's interview will take place on Saturday
D) Osama's interview will take place on Saturday.
E) Amin's interview will take place on Sunday.

Sol:

6 persons

Ali, Amin, Omer, Osama, Hamza, Saleem

Also

Saleem → Hamza

Hamza → Omer

Thursday	Friday	Saturday	Sunday
Saleem	Hamza	Ali	Amin, Omer

So, E is true

Example 8:

A chemist is preparing a nutrient using 8 different vitamins & minerals A, B, C, D, E, H, F and Z. According to the recipes, the following requirements apply to the use of ingredients:

1) **If B is used, both C and Z must also be used.**
2) **E & H must also be used together.**
3) **If 'C' is used, at least two of A, B & F must be used**
4) **C & H cannot be used together.**
5) **E, F & Z cannot all be used in the same nutrient**
6) **A, D & Z cannot all be used in same nutrient.**

Which of the following is a suitable combination for a nutrient?

a) **A, B, C, F**
b) **D, E, H, Z√**
c) **A, D, E, Z**
d) **C, D, E, F**
e) **E, H, F, Z**

Hints:

If B, C& Z must

E and H must

With C any two A, B, F

C, H, Z can not

A, D & Z cannot be used

Prepared by Azhar Afzal Mir

Calendar –related problems on days:

Question 1:

August 9th, 2013 is a Friday, what will be August 9th, 2014?

Sol:

Technique:

1) Add 1 day for normal year.
2) Add 2 days for leap year
3) To check, leap year divide by '4', if year is completely divisible, call leap year.

Question 2:

If it is Wednesday on 10th August 2011. What will be the 10th August 2012?

Question 3:

On 5th of September 1993, Nirma and Raja celebrated their anniversary Sunday. What will be the day on their anniversary in 1997?

Chapter 11: Probability/ Random Sampling

"The likelihood of occurrence of an event is called probability."

For any event 'A' probability can be expressed as follows:

$$P(A) = \frac{\textbf{No. of ways event can occur}}{\textbf{Total No. of possible outcomes(also called sample space)}}$$

Maximum probability is assumed to be "1" and minimum is "Zero".

i.e.

$$0 \leq P \leq 1$$

Probability 1=shows total certainty

Probability 0= show that something can't happen

Question of probability are normally asked from dice, cards, coins and other daily life problems. However, it becomes pretty simple to tackle all type of problems after learning it on dice, cards and coins etc.

Dice:

Example 1:

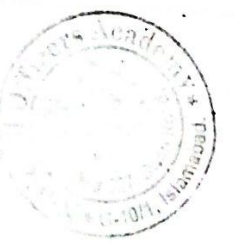
What is the probability of '5' when a dice is rolled?

Sol:

No. of ways '5' can occur = 1

Total no. of possible outcomes = 6

So



$$P(5) = \frac{1}{6}$$

Example 2:

Find the probability of getting an even result when a dice is rolled?

Sol:

As we know

$$Prob(E) = \frac{No.\ of\ ways\ even\ can\ occur}{Total\ possible\ outcomes}$$

So,

$$Prob(even\ result) = \frac{3}{6}\ (Beacuse\ 2, 4, 6\ are\ three\ even)$$

$$Prob(even) = \frac{1}{2}$$

FOR YOUR INFORMATION

Total cards = 52

Face cards = 12

Non-face cards = 40

Black cards=26, Red cards= 26

There are 4 major cards each having 13 others

Diamond 13 → King

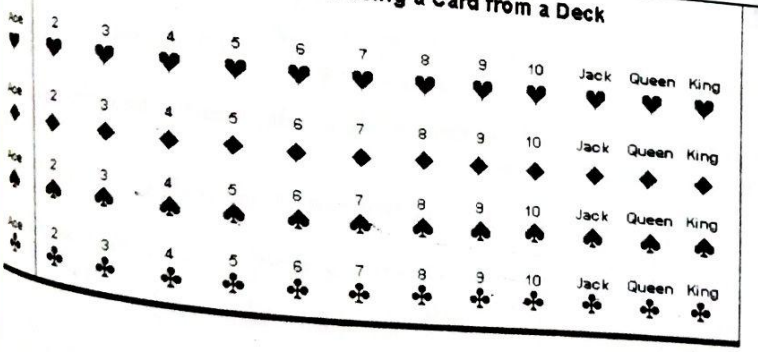
Heart 13 → Queen

Spade 13 → Jack

Club 13 → Ace

→ 9 others cards with numbers

Sample Space for Choosing a Card from a Deck



Example 3:

A card is randomly drawn from deck of 52 cards, what is the probability drawing a queen?

Sol:

$$Prob(E) = \frac{No.\ of\ ways\ event\ can\ occur}{Total\ possible\ outcomes}$$

Here event is queen:

$$Prob(Q) = \frac{4}{52}$$

$$= \frac{1}{13}$$

So,

$$Prob(Q) = \frac{1}{13}$$

Example 4:

A card is randomly drawn from deck of 52 cards, what is the probability of getting an Ace, King or Queen?

Sol:

$$Prob(Ace) = \frac{4}{52} = \frac{1}{13}$$

$$Prob(King) = \frac{4}{52} = \frac{1}{13}$$

$$Prob(Queen) = \frac{4}{52} = \frac{1}{13}$$

As the events are mutually exclusive; so, their probabilities are added up.

$$Prob(Q, A\ or\ K) = \frac{1}{13} + \frac{1}{13} + \frac{1}{13}$$

$$Prob(Q, A\ or\ K) = \frac{3}{13}$$

Mutually exclusive events:

"Two events are said to be mutually exclusive if occurrence of one event excludes the occurrence of the other event."

For instance, either Head or Tail can occur in case of coin.

Similarly, when a dice is rolled.Let's say two events 1 and 6. If 6 occur it excludes the occurrence of other event i.e. 1; so both the events 6 and 1 are mutually exclusive. Thus, their probabilities will be added up.

Example 5:

A number is selected at random from first thirty normal numbers. What is the chance that it will multiple of 3 or 13?

Sol:

$$Prob(E) = \frac{No.\ of\ ways\ an\ event\ can\ occur}{Total\ possible\ outcomes} \quad (1)$$

Multiple of 3 in first thirty natural numbers: 3, 6, 9, 12, 15, 18, 21, 24, 27,30.

Multiple of 30 in first thirty natural numbers: 13, 26

So, total no. of ways events (multiple of 3 and 13) can occur is 12

Hence, from (1)

$$Prob(Multiple\ of\ 3\ and\ 13) = \frac{12}{30}$$

$$Prob(Multiple\ of\ 3\ and\ 13) = \frac{2}{5}$$

Example 6:

When two dice are rolled, what is the probability that sum of outcome is 7?

Sol:

$$Prob(E) = \frac{No.\ of\ ways\ an\ event\ can\ occur}{Total\ possible\ outcomes}$$

Hence event is "sum 7" by adding the outcomes of two dice.

Dice 1	Dice 2
1	6
2	5
3	4
6	1
5	2
4	3

So, number of ways = 6

Hence

$$Prob(Sum\ 7) = \frac{6}{6 \times 6} = \frac{6}{36}$$

Here total outcomes are 36 i.e. 6×6 due to independent events.

Independent events in probability:

Events can be said independent if the occurrence or non-occurrence of an event does not influence the occurrence or non-occurrence of other event.

Example 7: (CSS- 2016)

Classification of blood groups is based on presence or absence of inherited antigenetic substances on the surface of red blood cells. In a survey of birth population the blood group distribution among 1000 people was as follows:

300 had blood group 'A', 325 had blood group 'B', 25 had '0' and 125 AB.Out of this group a person was selected at random. Calculate his probability of having blood group AB.

Sol:

$$Prob(E) = \frac{No.\ of\ ways\ an\ event\ can\ occur}{Total\ possible\ outcomes}$$

$$P(AB) = \frac{125}{1000}$$

$$= \frac{25}{200}$$

$$P(AB) = \frac{1}{8}$$

Example 8:

When two coins are tossed, what is the probability of heads on both the coins?

Sol:

Each coin has one tail and one head so total number of possible outcomes are two (Head and Tail).

Probability of occurring head on one coin $= \frac{1}{2}$

$$P(H) = \frac{1}{2}$$

Both the coins are independent so probabilities are multiplied

$$P(H,H) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$P(H,H) = \frac{1}{4}$$

Other method:

Sol:

Total No.of outcomes possible when two coins are tossed = 2 × 2 = 4 (independent events)

No. of ways of outcomes = {HH, HT, TH, TT}

Where

H = Head

T = Tail

No. of ways for heads on both coins = 1 (see among above four pair)

$$Prob(E) = \frac{No.\ of\ ways\ an\ event\ can\ occur}{Total\ possible\ outcomes}$$

$$Prob(HH) = \frac{1}{4}$$

Example 9:

One card is randomly drawn from a pack of 52 cards. What is the probability that the card drawn is face card?

Sol:

There are 10 face cards in the deck of 52 cards i.e. face cards mean the card on which face appears like that of King, Jack or Queen. There is no other card with such face in the deck.

As we know

$$Prob(E) = \frac{No.\ of\ ways\ an\ event\ can\ occur}{Total\ possible\ outcomes}$$

$$Prob(Face) = \frac{12}{52}$$

$$Prob(Face) = \frac{3}{13}$$

Example 10:

Johan draws a card from the pack of 52 cards. What is the probability that the card drawn is card of black suit?

Sol:

There are 26 black cards in the deck

So

$$Prob(black\ cards) = \frac{26}{52}$$

$$Prob(black\ cards) = \frac{1}{2}$$

Example 11:

A letter is chosen at random from the word "ASSASSINATION". What is the probability that it's a vowel?

Sol:

Total no. of letters in word = 13

Total no.of vowels in given word = 6

Prob (of getting vowel) $= \frac{6}{13}$

Example 12:

A bag contains 8 yellow, 7 blue and 6 black balls. What is the chance that ball is neither black nor yellow when only one ball is picked randomly?

Sol:

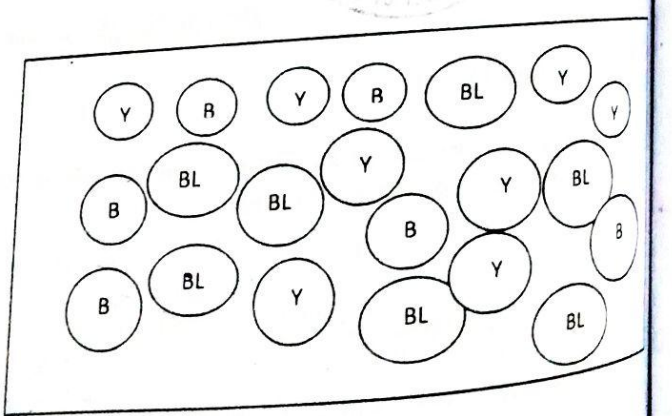
Total no.of balls $= 8 + 7 + 6 = 21$

No.of ways in which a ball can be selected which is neither yellow nor black $= 7$
(As there are only 7 balls which are neither black nor yellow).

$$Prob(E) = \frac{No.\ of\ ways\ an\ event\ can\ occur}{Total\ possible\ outcomes}$$

$$= \frac{7}{21}$$

$$= \frac{1}{3}$$



Y = Yellow

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B = Black

BL = Blue

Assignment

Question1

When three coins are tossed what is the probability of getting at most two tails?

Hint:

1)Total possibilities are 8 .You may check it

2^3

Possible outcomes — Coins

$= 2 \times 2 \times 2 = 8$

2) HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

Now solve it.

Question2

Cricket teams of Pakistan and India are playing one day match. What is the probability that Pakistan will not lose the match?

Solve it.

Arrangement problems

For the problem of arrangement in mathematics two techniques "combination" & "permutation"areused.

Combination:

"A collection of things in which order does not matter, called combination."

For example

Picking a team of 11 players for cricket out of 15 players does not require particular order.

Formula:

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Where '!' (Factorial) = Factorial is the product of descending order of total numbers

e.g. $3! = 3 \times 2 \times 1$

n = No. of things to choose from.

r = No. of things we choose.

Remember

1! = 1

0! = 1

Permutation:

"The collection of things where order matters, is called Permutation."

For example

Secret code of any safe = 246

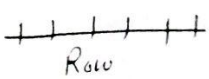
If you write 426, it's not work as order matters in this case.

Formula

$$nPr = \frac{n!}{(n - r)!}$$

Where n, r & ! assume the same meaning as in combination.

Example. 3 boys and 3 girls sit in a row of 6 chairs. In how many ways can they sit in alternate seats?



Soln

let 1st a girl sits on front chair

No. of ways = 3 (as three girls are there any one may be)

Now 2nd is boy = 3

Total possibilities = 3 X 3

Now girl sit (2 girls remaining)

ways = 3 X 3 X 2

Now boys next

ways = 3 X 3 X 2 X 2 X 1 X 1

ways = 36

Similarly if boys sits 1st again = 36

36 + 36 = 72

Chapter 12: L.C.M & H.C.F

L.C.M (Least common multiple)

By definition

"Product of common factors and uncommon factors is called L.C.M."

By common factor, we mean a number which divide commonly all given values. If the number is unable to divide any one of the given values, it is termed as uncommon factor. Followingexample will make this idea more clear:

Example 1

How many whole bricks $6\times12\times24\ cm^3$ will be suffieient to construct a solid cube of minimum size?

Sol:

(N.p): The word minimum hints us that problem is to be solved by L.C.M

So,

2	6, 12, 24
3	3, 6, 12
2	1, 2, 4
2	1, 1, 2
	1, 1, 1

(2 & 3 are common factors)

So, by definition

$$L.C.M = \textbf{Product of common factors and uncommon factors}$$

$$L.C.M = (2\times3)\times(2\times2)$$

Side of cube solid = 24 cm

$$\text{For volume of solid cube} = 24 \times 24 \times 24\ cm^3$$

Now, we can write logically as:

$$\text{Volume of solid cube} = \text{Volume of one brick} \times \text{No. of bricks}$$

$$\frac{\text{Volume of solid cube}}{\text{Volume of one brick}} = \text{No. of bricks}$$

$$\frac{24 \times 24 \times 24}{6 \times 12 \times 24} = \text{No. of bricks}$$

$$\text{No. of bricks} = 8$$

H.CF (Highest common factor)

By definition,

"The product of common factors only, is called H.C.F."

Example 2

A farmer wants to fence a triangular field, he plans to put a fencing post in each corner and place the other posts at equal distance along its sides. He wants the posts to be as far as possible. The sides of the field are 477 ft., 2412 ft. and 639 ft. long. How far apart will the posts be?

Sol:

Hint: (As far as possible)

3	477, 2412, 639
3	159, 804, 213
	53, 268, 71

As H.C.F is product of common factors only; so, not need to solve for uncommon factors.

Hence,

$$Distance = 3 \times 3$$

$$Distance = 9\ feet$$

Keep in mind

Product of two numbers = L.C.M × H.C.F

Following example will make you clear:

Example 3

L.C.M & H.C.F of two numbers are 150 and 30 respectively. If one number is 18, find the other number.

Sol:

As we know

Product of two numbers = L.C.M × H.C.F

$$18(2nd\ number) = 15 \times 30$$

$$2nd\ number = \frac{150 \times 30}{18}$$

$$2nd\ number = 250$$

Example 4

The lights blink after an interval of 6, 9 and 15 min respectively. If they blink together at 15pm, when will they blink together again (in contrast ,as late as possible for H.C.F)

Sol:

Here we have to find minimum, interval of time that light will blink again (L.C.M).

3	6, 9, 15
	2, 3, 5

As we know

$$L.C.M = (common\ factors) \times (uncommon\ factors)$$

$$= (3) \times (2 \times 3 \times 5)$$

$$= 90 \text{ min}$$

$$= 1 \text{ hr and } 30 \text{ min.}$$

The lights will blink at 6:30 pm together.

Example 5

Samia buys two off-cuts of a ribbon in a sale, one is 153 cm long, and other is 204 cm long. She cuts them so that she ends up with a number of pieces all with the same length. What is the greatest length of each piece?

Sol:

3	153, 204
17	51, 68

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H.C.F = 3×17

H.C.F = 51

Additionally, if you are asked to find no. of pieces then

$$No.\ of\ pieces = 3 + 4 = 7$$

Example 6

Find the H.C.F of 1.75, 5.6, 7

Sol:

Remove the decimal (use 1 replacing point and put no, of zeroes equal to the digits on the right side of decimal point.).

$$\frac{175}{100}, \frac{56}{10}, 7$$

In order to remove 100, multiply 100 together

$$\frac{175}{100} \times 100, \frac{56}{10} \times 100, 7 \times 100$$

$$175, 560, 700$$

Now, simply

5	175, 560, 700
7	35, 112, 140
	5, 16, 20

As

H.C.F= Product of common factors

$= (5)(7)$

$= 35$

Example 7

In a morning walk, three persons step-off together. Their step measure 75cm, 80cm and 90cm respectively. What is the minimum distance each should walk so that all can cover the same distance?

Sol:

5	75, 80, 90
2	15, 16, 18
2	15, 8, 9
2	15, 4, 9
2	15, 2, 9
3	15, 1, 9
3	5, 1, 3
5	5, 1, 1
	1, 1, 1

LCM

$$H.C.F = 5 \times 2 \times 2 \times 2 \times 3 \times 5$$

$$= 3600$$

So required minimum distance is 3600cm.

Example 8

Amina has two pieces of clothes one piece is 72 inches wide and other piece is 90 inches wide. She wants to cut both pieces into strips of equal width that are as wide as possible. How wide should she cut the strips?

Sol:

For largest possible equal width using H.F.C

2	72, 90
3	36, 45
3	12, 15
	4, 5

H.C.F or G.C.M (general common factor) $= 2 \times 3 \times 3 = 18$

So, Amina should cut each piece 18 inches.

Example 9. css 2017

It takes Ali 30 minutes to mark a paper.Aslam only needs 25 minutes to mark a paper. If the both start if they both start marking paper at 10:00 a.m. At what time they will finish marking at the same time?

Sol:

Same time means common time which conveys the meaningsof minimum or focus at single point.

5	30, 25
6, 5	

L.C.M = **5** × 6 × 5 = 150 minutes

150 minutes means two hours and thirty minutes.

So, from 10:00 a.m. it becomes 12:30 p.m.

Hence, they will finish marking at 12:30 p.m.

Assignment

Ben exercises every 12 days and Isabel every 8 days. Ben and Isabel both exercised today. How many days will it be until they exercise together again?

Practice problems:

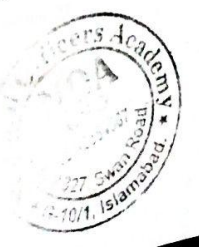
Question1: A radio station is having a promotion in which every 12th caller receives a free concert ticket and every 15th caller receives a limo ride. Which caller will be the first to win the ride? (60th caller)

Question 2: Cups are sold 5 to a package and plates are sold 10 to a package. If you want to have the same number of each item for a party, what is the least number of packages of each you need to buy? (10 of each)

Question 3: For a dinner party, Lara is creating individual servings of appetizers. She has 15 carrot sticks and 20 celery sticks. If she wants each serving to be identical, with no food left over, what is the greatest number of servings Lara can create?

Question 4: Liza has two pieces of thread, one 7 feet long and other 14 feet long. For a sewing project, she needs to cut them up to produce many pieces of thread that are all of same length, with no thread left over. What is the greatest length, in feet, that she can make them? (7)

Question 5: Grayson is thinking of a number that is divisible by both 5 and 4. What is the smallest possible number that Grayson could be thinking of? (20)



Chapter 13: Solution of Algebraic Equations

Linear equations:

"Equations with degree (maximum power of variable) one are called Linear equations"

For instance

$$x + y = 3$$

$$4x + 2y = 14$$

Quadratic equations:

"Equations with degree two are called quadratic equations."

For instance

$$9x^2 + 2y = 0$$

$$ax^2 + bx + c = 0$$

*Normally, linear equations are involved in general mathematics.

Developing a linear equation:

Sum of two numbers	⟶	x + y
Difference of two numbers	⟶	x – y
Two more than a number	⟶	x + 2
Five minus a certain number	⟶	5 - x
6 times a number	⟶	6 (x)
Two consecutive numbers	⟶	x, x + 1
Sum of three consecutive numbers	⟶	x, x+1, x+2

Product of two numbers is 20 ⟹ $(x)(y) = 20$

Two more than thrice a number ⟹ $3(x) + 2$

Example 1:

The cost of 3 tables and 8 chairs is 4350 rupees and cost of two tables and 5 chairs is 2800 rupees. Find the cost of one table and one chair.

Sol:

Let

$$\textbf{Cost of one table} = x$$

And

$$\textbf{Cost of one chair} = y$$

By given condition:

$$3x + 8y = 4350 \quad (1)$$

Similarly,

$$2x + 5y = 2800 \quad (2)$$

Multiplying (1) by "2" and (2) by "3"

$$2(3x + 8y) = 2(4350)$$

$$6x + 16y = 8700 \quad (3)$$

Also

$$3(2x + 5y) = (2800)3$$

$$6x + 15y = 8400 \quad (4)$$

Subtracting (4) from (3)

$$6x + 16y = 8700$$
$$-6x \pm 15y = -8400$$
$$1y = 300$$

Price of one chair = y = Rs.300

Using y=3 in (1)

$$3x + 8(300) = 4350$$
$$3x = 4350 - 2400$$
$$3x = 1950$$
$$x = \frac{1950}{3}$$
$$x = 650\ rupees$$

Price of one table = R.s.650

Practice problems:

Question 1: Sum of two numbers is 25. One of the number exceeds the other by 9. Find the numbers. (8, 17)

Question 2: Aran is 5 years younger than Ron. Four years later, Ron will be twice as old as Aran. Find their present ages. (1, 6 years)

Question 3: Two small pitchers and one large pitcher can hold 8 cups of water. One large pitcher minus one small pitcher constitutes 2 cups of water. How many cups of water can each pitcher hold? (2, 4)

Quadratic equation & quadratic formula

In general form a quadratic equation is the equation with degree (Maximum power of variables) two e.g.

$$ax^2 + bx + c = 0 \quad (1)$$

For such equations we can use quadratic formula to solve the unknown other than formulation techniques and completing square.

For equation (1)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Solve: $x^2 + 2x + 1 = 0$

Here

a = 1, b = 2, c=1

As we know

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(1)}}{2(1)}$$

$$\frac{-2 \pm \sqrt{4 - 4}}{2(1)}$$

$$= \frac{-2 \pm 0}{2} = \frac{-2}{2} = -1$$

$$x = -1$$

by 9.

twice

One cups

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Chapter 14: Age Problems Tricks And Solution Of Equations

Instructions:

Age related problems may be asked in present, past or future because it may be present age, past age or future age; other than these three ages related problems are not possible. Students try to develop directly mathematical equations from the statement and commit mistakes.Before developing equations directly, make yourself habitual to write present, past and future age in rough columns as in the following example.

In some problems only one out of three columns and in others all three columns may be used, but for aspirants with no mathematical background, it's recommended to be on one scheme of problem solving. So, always develop three columns; i.e. present, past and future whether all are used in the given problem or not.

Furthermore, you will be provided with some value of present age if not given let it to be 'x'. For past age, subtract the given years (e.g. for two years ago $x-2$) add given years for future (e.g. four years later $x+4$).

Following example will make it clear

Example 1:

Age of a man 7 years ago was 7 times the age of his son. Father's age after 3 years will be three times the age of son. Find their present ages.

Sol:

As trick

Present age	Past age	Future age
Let age of son = x	Son = x-7	Son = x +3
Let age of father = y	Father = y-7 (7 years ago)	Father = y+3 (Three years later)

Now it's easy to develop the equations without committing mistake.

Given that the age of man 7 years ago (past age) was 7 times the age of his son (Son's past age).

So.

$$y - 7 = 7(x - 7)$$

$$y - 7 = 7x - 49$$

$$7x - y = -7 + 49$$

$$7x - y = 42 \qquad (1)$$

According to second condition

$$y + 3 = 3(x + 3)$$

$$y + 3 = 3x + 9$$

$$3x - y = 3 - 9$$

$$3x - y = -6 \qquad (2)$$

Subtracting (2) from (1)

$$\begin{array}{r} 7x - y = 42 \\ -3x + y = +6 \\ \hline 4x = 48 \end{array}$$

$$x = \frac{48}{4}$$

$$x = 12 \text{ years} \qquad \textit{Age of som}$$

Using x = 12 in (2)

$$3(12) - y = -6$$

$$36 + 6 = y$$

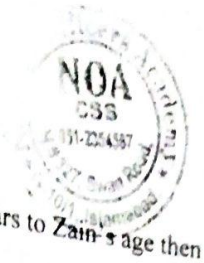
$$y = 42 \text{ years} \qquad \textit{Age of father}$$

Example 2:

Ali is 9 year older than Zain. In 7 years, the sum of their ages is equal to 93. Find their ages.

Sol:

Present age	**Future age**
Let age of Ali =x	Ali's age = x+ 7
Age of Zain = y	Zain's age= y+ 7



ording to first condition

is 9 year older than Zain, i.e. when we add 9 years to Zain's age then it comes equal to Ali's only.

$$Ali = Zian + 9$$

$$x = y + 9$$

year i.e. future age

$$x + 7 = y + 9 + 7$$

Original age of Ali Original age of Zain

$$x + 7 = y + 16$$

$$x - y = 16 - 7$$

$$x - y = 9 \qquad (1)$$

According to 2nd condition

$$(x + 7) + (y + 7) = 93$$

93.

$$x + 7 + y + 7 = 93$$

$$x + y = 93 - 14$$

$$x + y = 79 \qquad (2)$$

Adding (1) & (2)

$$2x = 88$$

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$x = 44$ *years* **Age of Ali**

Using x = 44 in (1)

$$44 - 9 = y$$

$y = 35$ *years* **Age of Zain**

Problems for practice

Question 1: 13 year ago Shabir's mother was 7 time as old as he was. She is now 48 years old. How many year old is Shabir now? (18)

Question 2: A man was 26 year old when his daughter was born. Now, he is three times as old as his daughter. How many year old is the daughter now? (13)

Question 3: If 5 years are added to a man's present age and the age is tripled, he will be 84. Find his present age. (23)

Question 4: A man's present age is 'x' years. If his age is 8 years, will be $\frac{4}{5}$ of what it will be in 2 years. Find his present age.

Question 5: If 12 years are added to $\frac{2}{3}$ the age of Ayesha, she will be three years older than today. What is her present age? (27)

Chapter 15: Rounding Of Numbers, Profit, Loss, Mental Abilities And Additional Logical& Analytical Solved Practice Problems

Number can be rounded by the following three rules:

1) If the dropped digit is less than 5 retained digit remains same.

Example 1:

Round the following number up to 1– decimal place?

75.23

By 1-decimal place, we mean one digit after decimal point is to be kept. So, **2** is the retained digit and **3** is dropped digit.

As dropped digit '3' is less than 5; hence

75.2

2) If the dropped digit is greater than 5 then retained digit is increased by one.

Example2:

Round the following number up to 2-decimal places:

8.1273

Here retained digit = 2

Dropped digit = 7

As, dropped digit is greater than 5

So,

8.13

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3) If the dropped digit is 5 then there are two possibilities for the retained digit:

- When retained digit is even, it remains same

For instance,

75.650

Rounded for 1 decimal place as:

75.6

- When the retained digit is odd, it is increased by one:

For instance,

64.350

Rounded up to one decimal place as:

64.4

Rounding of numbers in addition and subtraction

In adding or subtracting numbers, the number of decimal places retained in the answer should be equal to the smallest number of decimal place in any of the quantities being added or subtracted.

Following example will make the concept clear:

Example:

Add the following numbers and round off the answer:

72.1, 3.42, 0.003

Sol:

$$\begin{array}{r} 2.1 \\ 3.42 \\ 0.003 \\ \hline 5.523 \end{array}$$

As the least is one decimal place in the given numbers;

So, **75.5**

Practice problem

Round off the result after subtracting these two numbers

2.7543, 4.10

Rounding of numbers in division and multiplication:

In multiplication and division of numbers answer is rounded keeping in mind the least decimal/least no. of significant figures. Following examples will make it clear:

Example:

Solve and round off the result:

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336}$$

Sol:

$$= \frac{5.348 \times 10^{-2} \times 3.46 \times 10^{4}}{1.336}$$

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$$= \frac{5.348 \times 3.64 \times 10^2}{1.336}$$

Using scientific notation

$$= 1.45768982 \times 10^3$$

Applying rules of rounding

$$= 1.46 \times 10^3$$

Profit and loss formulae

In the case of profit and loss, percentage profit and percentage loss is asked to find out. For percentage profit on selling anything, following formula of %age profit is used:

$$\%age\ Profit = \frac{Profit}{Purchase\ price} \times 100$$

Where

Profit = sell price – Purchase price

Likewise

$$\%age\ Loss = \frac{Loss}{Purchase\ price} \times 100$$

Where

Loss = Purchased price – sell Price

Following example will help to understand the concept:

Example 1

A bicycle was purchased in 3450 rupees and sold in 3850 rupees. Find the percentage profit.

Sol:

As we know

$$\%age\ Profit = \frac{Profit}{Purchase\ price} \times 100$$

Where profit = sell price – Purchase price

Here, sell price = 3850

Purchase price = 3450

$$\%Profit = \frac{(3850 - 3450)}{3450} \times 100$$

$$= \frac{400 \times 100}{3450}$$

$$= 11.59$$

So, percentage profit ≅ 11.6% [" ≅ " shows approximation or Rounding]

Example 2

A boy purchased a book in 575 rupees and sold in 320 rupees. Find his percentage loss.

Sol:

As we know

$$\%\text{age Loss} = \frac{\text{Loss}}{\text{Purchase price}} \times 100$$

Where

Loss = Purchase price – sell price

So

$$\%ange\ Loss = \frac{(575 - 320)}{575} \times 100$$

$$= \frac{255}{575} \times 100$$

$$\%age\ Loss = \frac{4500}{115}\%$$

Practice problems:

Question 1: A trader gains 20% profit by selling a chair in 2700 rupees. Find the purchase price of chair. (2250).

Question 2: A shopkeeper get 20% loss when he sells a shirt in 960 rupees, on what price should be sold it to get 35% profit? (1620).

Mental abilities, additional logical and analytical solved practice problems)

1) **A told B "The girl I met yesterday was the youngest daughter of the brother-in-law of my friend's mother". How is the girl related to A's friend?**

Ans: Cousin

Explanation:

Daughter of brother-in –law……..means Niece

Mother's niece …….means cousin

So, the girl is the cousin of A's friend.

2) **A and B are young ones of C. If C is the father of A but B is not the son of C. How B and C are related?**

Ans: Daughter of father.

Explanation:

Chas two children A and B if A is his son B must be his daughter.

3) **A man pointing to a photograph says "The lady in the photograph is my nephew's maternal grandmother". How is the lady in the photograph related to the man's sister who has no other sister?**

Ans: Mother

Explanation: The lady is the grandmother of man's sister's son i.e. the mother of the mother of man's sister's son. i.e., the mother of man's sister.

4) **F is the brother of A, C is the daughter of A, K is the sister of F and G is the brother of C then who is the uncle of G?**

Ans: A

Explanation: C & G are children of A and F is the brother of A. so, F is the uncle of C and G

5) **A woman walking with a baby meets another woman and on being asked about her relationship with the boy, she says "My maternal uncle and his maternal uncles' maternal uncle are brothers". How is the boy related to the woman?**

Ans: Son

Explanation: Boy's maternal uncle will be brother of boy's mother. Maternal uncle of mother's brother and maternal uncle of lady are brother means lady is sister of mother's brother i.e. lady is the mother of the baby. So the boy is woman's son

6) **Six members of a family ABCDE and F are travelling together, B is the son of C but C is not the mother of B. A and C are married couple E is the brother of C. Dis daughter of A .F is the brother of B.How many male members are there in family?**

Ans: 4

Explanation: E (brother) C (Husband),A (wife), F & B (sons)D (daughter). A is the wife of C and their daughter is D. The remaining members are male members.

7) **1) A, B, C, D, E & F are six members of a family.**
2) One couple has parents and their children in the family.
3) A is the son of C and E is the daughter of A
4) D is the daughter of F who is the mother of E.

Which of the following pairs is the parents of the couple?

Ans: BC

Explanation: E is the daughter of A and F is the mother of E.So, A is the father of E and hence the husband of F.Now, D is the daughter of F. So, D & E are the daughters of A & F.Also, A is the son of C. Now, only B remains. Thus, B and C are the parents of A.Hence, B and C are the parents of the couple.

8) **A woman introduces a man as the son of the brother of her mother. How is the man related to the woman?**

Ans: Cousin

Explanation: Brother of mother .....Uncle

Uncle's son.....Cousin

9) **A told B that C is his father's nephew. D is A's cousin but not the brother of C. what is relationship is there between D and C?**

Ans: Sisters

Explanation: A has two cousins C and D. Since C is male, D must be female. And both are the nephew and niece of A's father.

10) **A, B, C, D, E, F & G are members of a family consisting of four adults and three children, two of whom F & G are girls. A and D are brothers and A is a doctor. E is an engineer married to one of the brothers and has two children. B is married to D and G is their child. Who is C?**

Ans: A's son.

Explanation: E is married to A or D.But B is married to D.Thus A, B, D & E are the four adults and C, F, and G are the three children of family. B and D have a child G. A & E have two children C & F .Now only F and g are girls. So C is a boy. Thus C is A's or E''s son.

11) **Pointing towards a person, a man said to a woman, "His mother is the only daughter of your father". How is the woman related to that person?**

Ans: Mother

Explanation: The only daughter of woman's father is herself.

So, the person is woman's son.i.e. The woman is person's mother.

12) **Pointing out to the girl a man said "My uncle is the uncle of this girl's uncle". How is man related to that girl?**

Ans: Father.

Explanation: The man's uncle and the uncle of girl is the same person. So, he may be the father of uncle of the girl.

13) In a family, there are six members A, B, C, D, E and F. A and B are married couple, A being the male member. D is the only son of C, who is the brother of A. E is the sister of D.B is the daughter-in-law of F, whose husband has died. How is E related to C?

Ans: Daughter.

Explanation:A is male and married to B.

So, A is the husband and B is wife. C is the brother of A. D is the son of C.

E, who is the sister of D will be the daughter of C.

B is the daughter-in-law of F whose husband has died means F is the mother of A.

Hence, E is the daughter of C.

14) If S is brother of N, the sister of N is M, the brother of P is J and the daughter of S is P then who is the uncle of P?

Ans:P

Explanation: As J and P are real brothers and their father is S whose brother is N, so N is the uncle of J & P.

15) There are 6 persons A, B, C, D, E & F .C is the sister of F.B is the brother of E's husband. D is the father of A and the grandfather of F. There are two fathers, three brothers and a mother in the group. Who is the mother?

Ans: E

Explanation:D is the father of A and grandfather of F. So, A is the father of F. Thus, D and A are two fathers. C is the sister of F. so, C is the daughter of A. Since there is the only mother, it is the evident that E is the wife of A and hence the mother C and F.

16) A girl introduced by a boy as the son of the daughter of the father of her uncle. The boy is girl's?

Ans: Brother.

Explanation: Daughter of uncle's father .....uncle's sister.....Mother; Mother's son..... Brother.

17) **'A' is the brother of B and k, D is the mother of B and E is the father of A. Which of the following statement is not definitely true?**

Ans:A is the father of k

Explanation:A, B and K are brother and sisters. E & D may be husband and wife. So, A is the son of E and D both.

18) **There are 6 persons A, B, C, D, E and F .Cis the sister of F. B is the brother of E's husband. D is the father of A and grandfather of F. There are two fathers, three brothers and mother in group. Who is E's husband?**

Ans: A

Explanation: D is the father of A and grandfather of F. So, A is the father of F. Thus, D and A are two fathers. C is the sister of F. so, C is the daughter of A. Since there is the only mother, it is the evident that E is the wife of A and hence the mother C and F.

So, B is the Brother of A. There are three brothers.

So, B is the brother of C

Hence, A is E's husband.

19) **Pointing to a photograph, a man said "I have no brother or sister but that man's father is my father's son". Whose photograph was it?**

Ans: His son's

Explanation: Since the narrator hos no brother, his father's son is himself. So the man who is talking is the father of the man in photograph. Thus, the man in the photograph is his son.

20) **A is the mother of B and C, if D is husband of C. what is A to D?**

Ans: Mother-in-law

Explanation: A is the mother of C and C is wife of D.

So, A is mother-in-law of D, or D is son-in-law of A.

21) There are 6 persons A, B, C, D, E and F .C is the sister of F, B is the brother of E's husband. D is the father of A and grandfather of F. There are two fathers, three brothers and mother in group. Which of the following is a group of brothers?

Ans: ABF

Explanation: D is the father of A and grandfather of F. So, A is the father of F. Thus, D and A are two fathers. C is the sister of F. so, C is the daughter of A. Since there is the only mother, it is the evident that E is the wife of A and hence the mother C and F.

So, B is the Brother of A. There are three brothers.

So, B is the brother of C. so three brothers are A,B,F.

22) A man is said to a lady "your mother's husband's sister is my aunt". How is the lady related to the man?

Ans: Sister

Explanation: Lady's mother husband……..Lay's father

Lady's father's sister …..Lady's aunt

So, lady's aunt is man's aunt and therefore lady is man's sister.

23) A family consists of six members P, Q, R, X, Y and Z. Q is the son of R but R is not mother of Q.P and R are married couple .Y is the brother of R . X is the daughter of P.Z is brother of P. Who is the father of Q?

Ans: R

Explanation: Q is the son of R but R is not the mother. So R is father of Q. P is married to R.So, P is the wife of R and mother of Q. X is the daughter of P and

hence of R and so, she is the sister of Q. Y is the brother of R and Z is the brother of P. R is the father of Q.

24) **A, B, C, D, E and F are members of a club. There are two married couples in the group. A is the brother of D's husband .C is the president of Women's Association. F is a Sitar player, and Bachelor. B's wife is not a member of the club. Four of them belong to the same family and F are colleagues in the club. How is F related to B?**

Ans: It is not possible to determine.

Explanation: According to the data ,there are two female members D and C of the club and data is not sufficient regarding B and F.

25) **If X is the brother of the son of Y's son, how is X related to Y?**

Ans: Grandson

Explanation:

Son of Y's son……..Grandson

Brother of Y's grandson…..Y's grandson.

So X is Y's grandson.

26) **Introducing a man, a woman said "His wife is the only daughter of my father". How is the man related to the woman?**

Ans: Husband.

Explanation: Only daughter of my father…… Myself

So, the man is woman's husband.

27) **A family consists of six members P, Q, R, X, Y and Z. Q is the son of R but R is not mother of Q.P and R are married couple .Y is the brother of R . X is the daughter of P.Z is brother of P. How many children does P have?**

Ans: 2

Explanation: Q is the son of R but R is not the mother. So R is father of Q. P is married to R.So, P is the wife of R and mother of Q. X is the daughter of P and hence of R and so, she is the sister of Q. Y is the brother of R and Z is the brother of P. Hence, Q is the son of P and X is the daughter of P.So, P has two children.

28) A family consists of six members P, Q, R, S, T and U. There are two married couples.Q is a doctor and the father of T.U is grandfather of R and is a contractor. S is grandmother of T and is housewife. There is one doctor, one contractor, one nurse, one housewife and two students in the family. What is the profession of P?

Ans: Nurse

Explanation: Q, the Doctor is the father of T. S, the Housewife is the grandmother of T and hence the mother of Q. Since there are only two married couples one being that of Q, the grandfather of R i.e. U must be married to S

Thus R and T will be both children of Q and these must be students. So, P who remains shall be the wife of Q and she alone can be the nurse.

29) Pointing to a photograph of a boy Suresh said "He is the son of the only son of my mother". How is the Suresh related to that boy

Ans: Father

Explanation: The boy in the photograph is the only son of the son of Suresh's mother i.e., The son of Suresh Hence, Suresh is the father of boy.

30) If A+B means A is the mother of B; A-B means A is the brother of B; A%B means A is the father of B and A×B means A is the sister of B , which of the following shows that P is the maternal uncle of Q?

Ans:P-M+N×Q

Explanation:

P-M.....P is the brother of M

M+N..... M is the mother of N

N×Q..... N is the sister of Q

Therefore P is the maternal uncle of Q

31) **If A+B means A is the brother of B; A-b means A is the sister of B and A×B means A is the father of B. Which of the following means that C is the Son of M?**

Ans: M×N – C + F

Explanation:

M×N..... M is the father of N

N- C ..... N is the sister of C

C+F..... C is the brother of F

Hence, M is the father C or C is the son of M

32) **Introducing a boy, a girl said "He is the son of the daughter of the father of my uncle". How is the boy related to girl?**

Ans: Brother

Explanation: The father of boy's uncle .....the grandfather of boy and daughter of the grandfather ... sister of father.

33) **Pointing to a photograph Lata says "He is the son of the only son of my grandfather". How is the man in the photograph related to Lata?**

Ans: Brother

Explanation: The man in the photograph is the son of the only son of Lata's grandfather i.e. , the man is the son of Lata's father. Hence, the man is the brother of Lata.

34) **If A+B means A is the brother of B; A×B means A is the son of B; and A%B means B is daughter of A then which of the following means M is the maternal uncle of N?**

Ans: None of these.

Explanation: Because the sex of O is not known.

35) **If A+B means A is the father of B; A-B means A is the brother of B; A%B means A is the wife of B and A×B means A is the mother of B, which of the following shows that M is the maternal grandmother of T?**

Ans:M×N%S+T

Explanation:

M×N ……..M is the mother of N

N%S ……..Nis the wife of S

S+T…….S is the father of T

Hence, M is maternal grandmother of T

36) **Pointing to a photograph,Bajpai said "He is the son of the only daughter of the father of my brother". How Bajpai is related to the man in the photograph?**

Ans: Maternal uncle

Explanation: The man in the photo is the son of the sister of Bajpai. Hence, Bajpai is the maternal uncle of the man in the photograph.

37) **Deepak said to Nitin "That boy playing with the football is the younger of the two bothers of the daughter of my father's wife". How the boy playing football isrelated Deepak?**

Ans: Brother

Explanation:

Father's wife…….means mother. Hence, the daughter of the mother means sister and sister's younger brother means brother. Therefore, the boy is the brother of Deepak.

38) **Pointing a photograph x said to his friend Y "she is the only daughter of the father of my mother". How X is related to the person of photograph?**

Ans: Son

Explanation: The only daughter of the father of X's mother means mother of X.

Hence X is the son of the lady in photograph.

39) **If A+B means A is the sister of B; A×B means A is the wife of B, A%B means A is the father of B and A-B means A is the brother of B. Which of the following means T is the daughter of p?**

Ans:P+Q%R-T+S

Explanation:

P×Q.....P is the wife of Q

Q%R...Q is the father of R

R-T.....R is the brother of T

T+S ...T is the sister of S

Therefore, T is the daughter of P.

40) **Pointing to a woman, Abhijit said, "Her daughter is the only daughter of my brother". How is the woman related to Abhijit?**

Ans: Mother.

Explanation:

Daughter of Abhijit's brother means niece of Abhijit. Thus the granddaughter of the woman is Abhijit's niece.

Hence, the woman is the mother of Abhijit.

41) **Amit said "This girl is the wife of the grandson of my mother". How is Amit related to the girl?**

Ans: Father-in-law

Explanation: The girl is the wife of grandson of Amit's mother.i.e. , The girl is the wife of son of Amit. Hence, Amit is the father –in-law of girl.

42) A and B are children of D. Who is the father of A? To answer this question which of the statements (1) and (2) is necessary?

C is the brother of A and son of E.

F is the mother B

Ans: Only (2)

Explanation:

A and B are children of D.

From (1), C is the brother B and son of E.

Since, the sex of D and E are not known. Hence (1) is not sufficient to answer the question.

From (2) F is the mother of B.Hence, F is also the mother of A .Hence D is the father of A.Thus C is sufficient to answer the question.

43) Pointing towards a man a woman said, "His mother is the only daughter of my mother". How is the woman related to the man?

Ans: Mother.

Explanation: Only daughter of my mother means myself.

Hence, the woman is the mother of the man.

44) Introducing Sonia, Amir says "She is the wife of only nephew of only brother of my mother". How Sonia is related to Amir?

Ans: Wife.

Explanation:

Brother of mother means maternal uncle. Hence only nephew of Amir's maternal uncle means Amir himself. Therefore Sonia is the wife of Amir.

45) **If A+B means A is the brother of B; A%B means A is the father of B and A×B means A is the sister of B. Which of the following means M is uncle of P?**

Ans: M+K%T×P

Explanation:

M+K.....M is the brother of K

K%T.....K is the father of T

T×P.....T is the sister of P.

Therefore, K is the father of P and M is the uncle of P.

46) **Pointing to Varman, Madhav said "I am the only son of the one of the sons of his father". How Warman related to Madhav?**

Ans: Father or Uncle

Fathere

Explanation:

Madhav is the only son of one of the sons of Varman's father means Either Varman is the father or Uncle of Madhav.

47) **Introducing a woman, Shashank said "She is the mother of the only daughter of my son". How that woman is related to Shashank?**

Ans: Daughter-in-law

Explanation:

The woman is the mother of Shankar's granddaughter. Hence, the woman is the daughter-in-law of Shankar

48) If A+B means B is the brother of A; A×B means B is the husband of A; A-B means A is the mother of B and A%B means A is the father of B, which of the following relation shows that Q is the grandmother of T?

Ans: Q-P+R%T

Explanation: Q-P.....means Q is the mother of p

P+R.....means R is the brother of P

Hence ...Q is the mother of R

R%T ....R is father of T

Hence, Q is the grandmother of T

49) A3P means A is the mother of P

A4p means A is the brother of P

A9P means A is the husband of P

A5P means A is the daughter of P

Which of the following means K is the mother in law of M?

Ans: M9N5K3J

Explanation:

M9N.... M is the husband of N

N5K:... N is the daughter of K

Hence....M is the son in law of K

K3J.... K is the mother of J

Hence, K is the lady

Hence, K is the mother-in-law of M

50) **Pointing to a photograph Anjali said, "He is the son of the only son of my grandfather". How is the man in the photograph related to Anjali?**

Ans: Brother

Explanation:

The man in the photograph is the son of Anjali's grandfather's son i.e., the son of Anjali's father. Hence the boy is the brother of Anjali.

51) **Pointing to a person, Deepak said, "His only brother is the father of my daughter's father. How is the person related to Deepak?**

Ans: Uncle

Explanation:

Father of Deepak's daughter father ....Deepak's father.

Hence the person is the brother of Deepak's father.

Therefore, the person in the uncle of Deepak.

52) **P is the mother of K; K is the sister of D; D is the father of J. How is P related to J?**

Ans: Grandmother

Explanation:

P is the mother of K

K is the sister of D

D is the father of J.

Therefore, J is the nephew or niece of K and P is the Grandmother of J.

53) **If A $ B means A is the brother of B; A @ B means A is the wife of B; A#B means a is the daughter of B and A*B means A is the father of B, which of the following indicates that U is the father-in-law of P?**

Ans: P @ q $ T # U * W

Explanation: P @ Q-> P is the wife of Q... (1)

Q $ T..... Q is the daughter of U... (2)

T # U ..... T is the daughter of U

Hence, .Q is the son of U.... (3)

U*W .....U is the father of W.

From (1) and (3), U is the father-in-law of P.

54) Introducing a man, a woman said, "He is the only son of the mother of my mother. How is the woman related to man?

Ans: Niece

Explanation:

The man is the only son of the mother of the woman .Hence, the man is the maternal uncle of the woman.so, and the woman is the niece of the man.

55) Pointing to Gopi, Nalni says, "I am the daughter of the only son of his grandfather". How Nalni is related to Gopi?

Ans: Sister.

Explanation: Nalni is the daughter of the only son of Gopi's grandfather. Hence it's clear that Nalni is the sister of Gopi.

56) A's son B is married with C whose sister D married to E the brother of B.

How D is related to A?

Ans: Daughter's-in-law

Explanation:

Since E is the brother of B

Therefore, A is the father of E

But D is the daughter-in-law of A.

57) Pointing to the lady a person said "The son of her only brother is the brother of my wife". How is the lady related to the person?

Ans: Sister of father-in-law.

Explanation:

Brother of person's wife-> brother-in-law of the person. Hence the son of lady's brother is brother-in-law of the person. Therefore, the brother of the lady is the father-in-law of the person of the person. Hence, the lady is the father-in-law.

58) **B5D means B is the father of D.**

B9D means B is the sister of D.

B4D means B is the brother of D.

B3D means B is the wife of D.

Which of the following means F is the mother of K?

Ans: F3M5K

Explanation:

F3M-> F is the wife of M.

M5K-> M is the father of K.

Therefore, F is the mother of K.

59) **A $ B meansA is the father of B;A#B means A is the sister of B;A*B means A is the daughter of B and A @B means A is the brother of B. which of the following indicates that M is the wife of Q?**

Ans: Q $ R @ T * M

Explanation:

Q $ R->Q is the father of R

R @ T->R is the brother of T

Hence, ->Q is the father of T

T*M->T is the daughter of M

Hence, ->M is the mother of T

Hence, M is the wife of Q.

60) If A $ B means A is the brother of B;B*C means B is the son of C; C @ D means C is the wife of D and A # D means A is the son of D. how C is related to A ?

Ans: Mother

Explanation:

A $ B ->A is the brother of B
B * C -> B is the son of C
Hence, -> A is the son of C

C @ D->C is the wife of D

Hence, ->C is the mother of A

61) Pointing to a girl Sandeep said, "She is the daughter of the only sister of my father". How is the Sandeep related to the girl?

Ans. Cousin

Explanation:

The girl is the daughter of the sister of Sandeep's father. Hence, the girl is the cousin or Sandeep is the cousin of the girl.

62) Pointing to a boy in the photograph Reena said, "He is the only son of the only child of my grandfather." How Reena is related to that boy?

Explanation:

The boy in the photograph is the only son of Reena's grandfather's only son; i.e., the boy is the only son of Reena's father

63) If M x N means M is the daughter of N; M+N means M is the father of N; M % N means M is the mother of N and M- N means M is the brother of N then P %Q+R-T x K indicates which relation of P to K?

Ans: none of these

Explanation:

P % Q -> P is the mother of Q

Q + R -> Q is the father of R

R - T -> R is the brother of T

Hence, -> Q is the father of T

T x K -> T is the daughter of K

Hence, -> Q is the husband of K.

Therefore, P is the mother-in-law of K.?

64) **If P+ Q means P is the brother of Q; P x Q means of P is the father of Q and P -Q means P is the sister of Q, which of following relations shows that *I is the niece of K?***

Ans: K + Y x I -Z

Explanation:

K + Y -> K is the brother of Y

Y x I -> Y is the father of I

Hence, -> K is the uncle of I

And I - Z -> I is the sister of Z

Hence, -> I is the niece of K.

65) **Pointing towards a girl, Abhisek says, "This girl is the daughter of only a child of my father". What is the relation of Abhisek's wife to that girl?**

Ans: mother

Explanation:

Only the child of my father means 'Abhisek' himself. This means the girl is the daughter of Abhisek. Hence, Abhisek's wife is the other of the girl.

66) **A is the son of C; C and Q are sisters; Z is the mother of Q and P is the son of Z, which of the following statements is true?**

Ans: P is the maternal uncle of A

Explanation:

C and Q are sisters and A is the son of C. hence, C is the mother of A or Z is the mother of Q.

Logical reasoning

Directions to solve:

Read the question carefully and write the correct answer:

1. **Four defensive football players are chasing the opposing wide receiver, who has the ball. Qaiser is directly behind the ball of carrier. Javed and Baber are side by side behind Qaiser. Zia is behind Javed and Baber. Qaiser tries for the tackle but misses and falls. Baber trips. Which defensive player tackles the receiver?**

   Ans: Javed

   Explanation: After all switching was done, Javed was directly behind the receiver. Qaiser and Baber had fallen. Zia remained in the rear

2. **In a four day-period Monday through Thursday each of the following temporary office workers worked only one day, each different day. MS. Johnson was scheduled to work on Monday, but she traded with Mr. Carter, who was originally scheduled to work on Wednesday .Ms. Falk traded with Mr. Kirk, who was originally scheduled to work on Thursday. After all switching was done, who worked on Tuesday?**

   Ans: Mr.Kirk.

   Explanation: After all the switches were made, Mr.Kirk worked on Tuesday .Mr. Carter worked on Monday, Ms. Johnson on Wednesday and Ms. Falk on Thursday.

3. **Four people witnessed a mugging. Each gave a different description of the mugger. Which description is probably right?**

   Ans: He was tall, thin and middle aged.

Explanation: Tall, thin and middle aged are the elements of description repeated most often and are.

4. **Ms. Forest likes to let the students chose who their partner will be; however, no pair of students may work together more than seven class periods in row. Adnan and Baxter have studied together seven class periods in a row. Carter and Dennis have worked together three class periods in a row. Carter does not want to work with Adnan. Who should be assigned to work with Baxter?**

Ans: Carter

Explanation: Baxter should be assigned to study with Carter. Baxter cannot be assigned with Adam, because they have already been together for seven class periods. If Baxter is assigned to work with Dennis that would leave Adam with Carter, but Carter does not want to work with Adam

5. **At the baseball game, Henry was sitting in seat 253.Marla was sitting to the right of Henry in seat 254. In the seat to the left of Henry was George. Inez was sitting to the left of George. Which seat is Inez sitting in?**

Ans:251

Explanation: If George is sitting at Henry's left, George's seat is 252. The next seat to the left, then is 251

6. **As they prepare for the state championship, one gymnast must be moved from level two team to the level 1 team. The coaches will move the gymnast who has won the biggest prize and who has the most experience. In the last competition, Roberta won a bronze medal and has competed seven times before. Jamie has won a silver medal and has competed a fewer times then Roberta. Beth has won a higher medal than Jamie and has competed more times than Roberta. Michele has won a bronze medal and it is her third time competing. Who will be moved to level 1 team?**

Ans: Beth.

Explanation: Beth won the biggest prize, described as higher medal than Jamie's which we have been told was a silver medal. Roberta and Michele both won bronze medals, which are lower ranking medals than silver. Beth is also described as having competed more times than Roberta who has

competed seven times. Jamie is described as having competed fewer times than Roberta and Michele has competed three times. Therefore, Beth has competed more times than the others and has won the biggest prize to date.

7. **Four friends in the sixth grade were sharing a pizza. They decided that the oldest friend would get the extra piece .Randy is two months older than Greg, who is three months younger than Ned. Kent is one month older than Greg. Who should get an extra piece of pizza?**

Ans: Ned.

Explanation: If Randy is two month older than Greg then Ned is three months older than Gregand one month older than Randy.Kent is the younger than both Randy and Ned. Ned is the oldest.

8. **The high school math department need to appoint a new chairperson, which will be based upon seniority. MS. West has less seniority than Mr. Temple, but more than Mr. Brody. Mr. Rhodes has more seniority than Ms. West, but less than Ms. Temple .Mr. Temple does not want the job. Who will be the new math department chairperson?**

Ans: Mr. Rhodes.

Explanation: Mr. Temple has the most seniority but he does not want a job next is Mr. Rhodes, who has more seniority than Ms. West and Mr.Brody.

9. **Danielle has been visiting friends in Ridge-wood for the past two weeks. She is leaving tomorrow morning and her flight is very early. Most of her friends live fairly close to the airport. Madison lives in ten miles away. France lives five miles away .Samantha, seven miles. Alexis is farther away than Frances, but closer than Samantha, Approximately how far away from the airport is Alexis?**

Ans: Six miles.

Explanation: Alexis is farther away than Frances, who is five miles away, and closer than Samantha, Who is seven miles away.

10. **Nurse Kemp has worked more night shifts in a row than Nurse Roger, who has worked five. Nurse Miller has worked fifteen nights' shifts in a row,**

more than Nurse Kemp and Rogers combined. Nurse Calvin has worked eight night shifts in a row, less than Nurse Kemp. How many night shifts in a row has Nurse Kemp worked?

Ans: Nine.

Explanation: Kemp has worked more shifts in a row than Nurse Calvin; Therefore Kemp has worked more than eight shifts. The number of Kemp's shifts plus the number of Roger's shifts (five) cannot equal fifteen or more, the number of Miller's shifts. Therefore, Kemp has worked nine shifts in a row (5 + 9 = 14).

11. **Children are in pursuit of dogs whose leash has broken. James is directly behind the dog. Ruby is behind James. Rachel is behind Ruby. Max is ahead of the dog walking down the street in the opposite direction. As the children and dog pass, Max turns around and join the pursuit. He runs in behind Ruby. James runs faster and is alongside the dog on the left. Ruby runs faster and is alongside the dog on the right. Which child is directly behind the dog?**

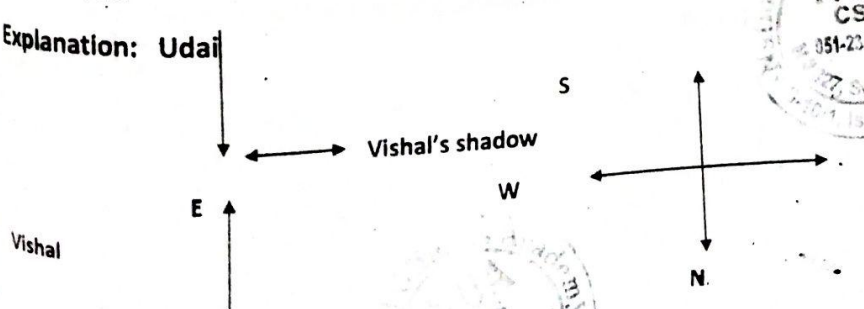
Ans: Max

Explanation: After all the switches were made, Max is directly behind the dog. James is alongside the dog on the left, Ruby is alongside the dog on the right, and Rachel is behind of Max.

12. **One morning Udai and Vishal were talking to each other face to face at a crossing. If Vishal's shadow was exactly to the left of Udai, which direction was Udai facing?**

Ans: North.

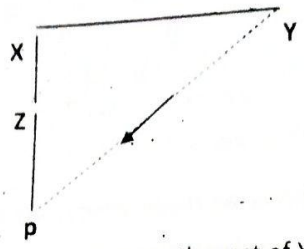
Explanation: Udai



13.Y is the East of X which is in the North of Z.If P is in the south of Z, then in which direction of Y, is P?

Ans: None of these.

Explanation:

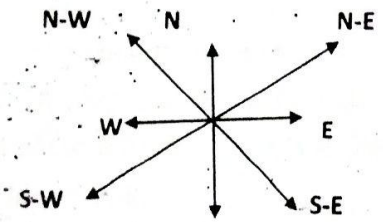


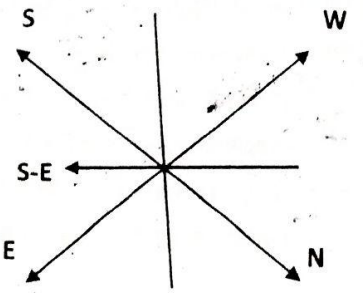
p is in south-west of y

14.If the south-East becomes North, North-East becomes West and so on, what will West become?

Ans: South-East.

Explanation:



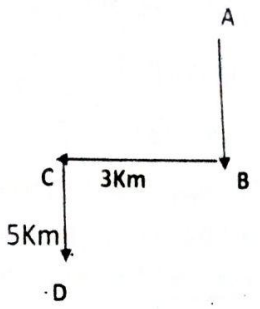


It is clear from the diagram that the new name of West will become South East.

15.A man walks 5Km toward South and then turns to the right. After walking 3 Km he turns to the left and walks 5Km. Now in which direction is he from the starting place?

Ans: South- West

Explanation:



Hence required directions is South-West:

16. Rahul put his time place on the table in such a way that at 6 P.M. hour hand points to North. IN which direction the minute hand will point at 9:15 P.M.?

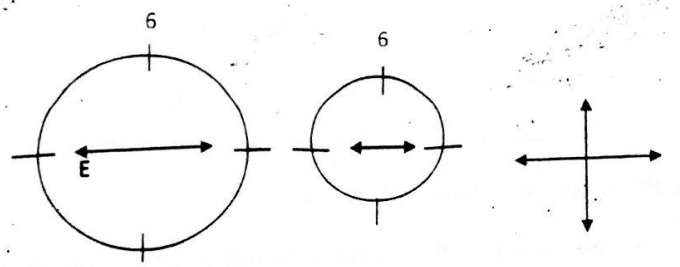
Ans: West

Explanation:

N

399 W

1212 S

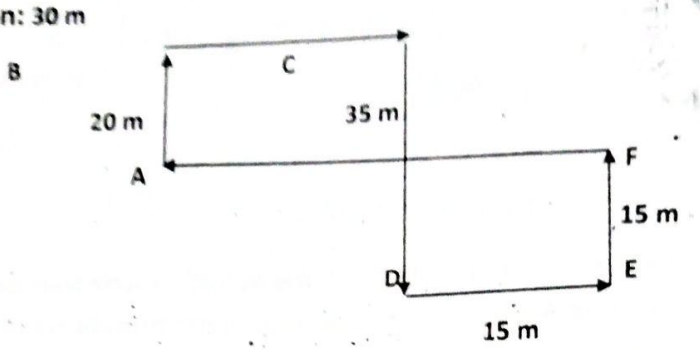


At 9:15 PM, the minute hand will point towards west

17. Rasik walked 20 m towards north. Then he turned right and walks 30 m. Then he turns right and walks 35 m. Then he turns left and walks 15 m. Finally he turns left and walk 15m. In which direction and how many meters is he from the starting position?

Ans: 45 m east

Explanation: 30 m



$$Required\ distance = AF$$

$$= 30 + 15$$

$$= 45\ m$$

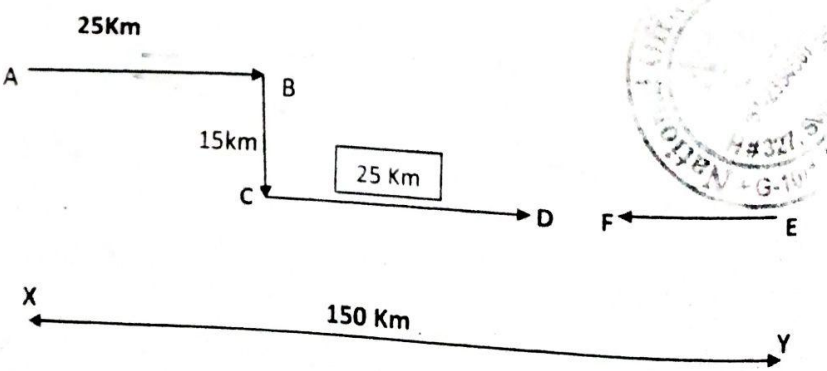
From the above diagram, F is in the East direction from A

Hence the required answer is 45 m east.

18. Two cars starts from the opposite places of a main road, 150 Km apart. First car runs for 25 Km and takes a right turn and then runs 15Km. It then turns left and then runs for another 25Km and then takes the direction back to reach the main road. In the meantime, due to minor break down the other cars has run only 35 Km along the main road. What would be the distance between two cars at this point?

Ans: 65 Km

Explanation:

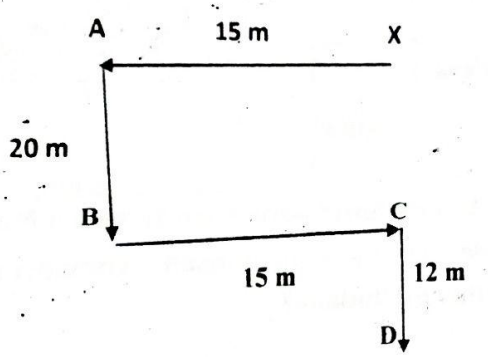


$$\text{Required distence} = DF$$
$$= 150 - (25 + 25 + 35)$$
$$= 150 - 85$$
$$= 65 \text{ Km}$$

19. Starting from the point X, Jayant walked 15 m towards west. He turned left and walked 20 m. He then turned left and walked 15 m. After this he turned to his right and walked 12 m. How far and in which directions is now Jayant from X?

Ans: 32 m, south

Explanation:

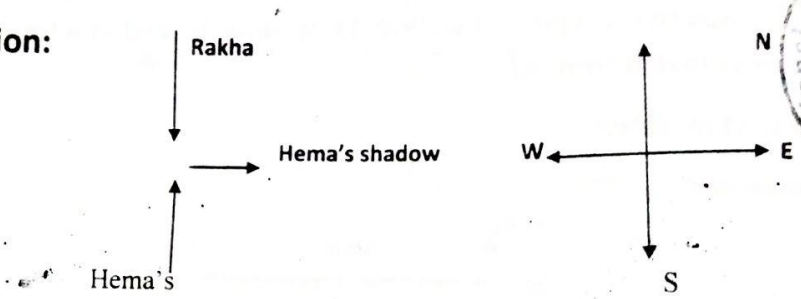


$$\textbf{\textit{Required distance}} = 20 + 12$$
$$= 32\ m\ in\ south\ distance$$

20. One evening before sunset Rekha and Hema were talking to each other face to face. If Hema's shadow was exactly to the right of Hema, which direction was Rekha facing?

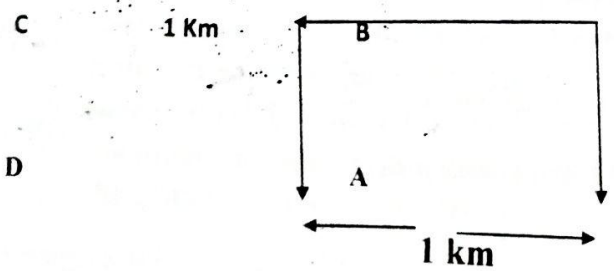
Ans: south

Explanation:



21. A boy rode his bicycle northward, then turned left and rode 1 km and again turned left and rode 2 km.He found himself 1 km west of his starting point. How far did he ride northward initially?

Ans:2 km

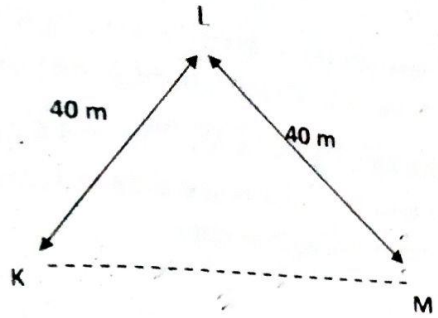


The boy rode 2 Km. Northward

22. K is 40 m south-west of L. if M is 40 m south-east of L, then M is in which direction of K?

Ans: East

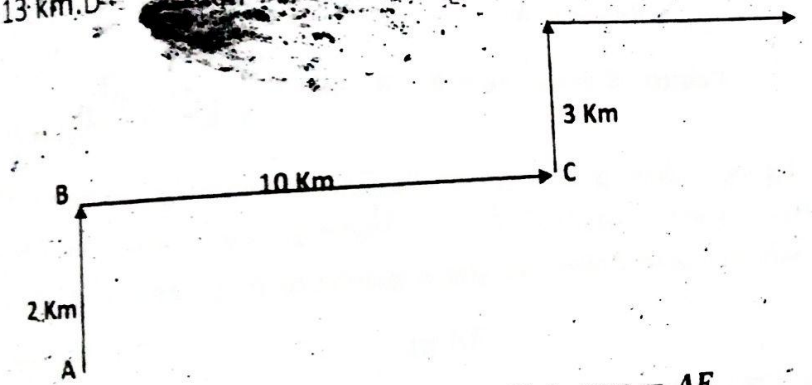
Explanation:



Hence M is in the East of K.

23. A man walks 2 km towards north. Then he turns to east and walks 10 km. after this he turns to north and walks 3 km. Again, he turns towards east and walks 2 km. How far is he from the starting point?

Ans: 13 km. D



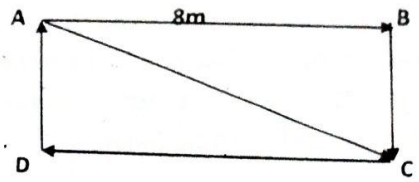
$$\textbf{Required distance} = AE$$

$$= \sqrt{5^2 + 12^2}$$

$$= 13\ Km$$

24. The length and breadth of a room are 8 m and 6 m respectively. A cat runs along all the four walls and finally along a diagonal order to catch a rat. How much total distance is covered by the cat?

Ans: 38



$$\textit{Required distance} = 8 + 6 + 8 + 6 + \sqrt{8^2 + 6^2}$$

$$= 28 + \sqrt{100}$$

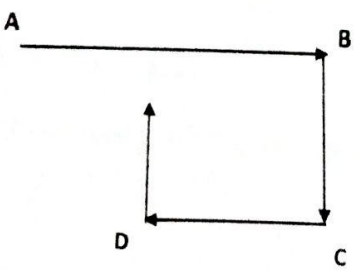
$$= 28 + 10$$

$$38\ m$$

25. Once morning Sujata started to walk towards the sun. After covering some distance she turned to right then again to the right and after covering some distance she again turns to the right. Now in which direction is she facing?

Ans: North

Explanation:

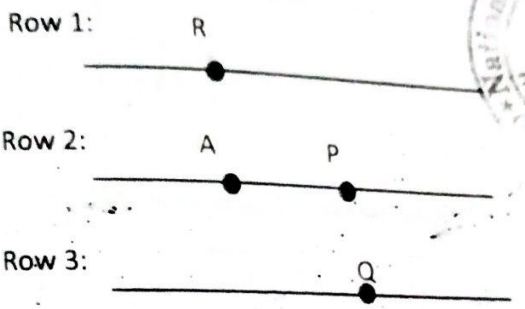


Hence finally Sujata will face towards North.

26. Some boys are sitting in three rows all facing north such that A is in the middle row. P is just to the right of A but in the same row. Q is just behind of P while R is in the north of A. in which direction of R is Q?

Ans: South-East

Explanation:



Q is in South-East of R

27. One morning after sunrise, Vimal started to walk. During this walking he met Stephen who was coming from opposite direction. Vimal watch that the shadow of Stephen to the right of him (Vimal). To which direction Vimal was facing?

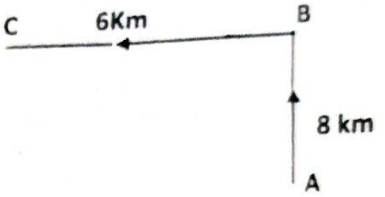
Ans: South

Explanation: Sun rises is in the East. So the shadow of a man will always falls towards the West. Since the shadow of Stephen is to the right of Vimal. Hence Vimal is facing towards south.

28. Golu started from his house towards North. After covering a distance of 8 km. he turned towards left and covered a distance of 6 km. what is the shortest distance now from his house?

Ans: 10 km.

Explanation:



$$Required\ distance = AC$$

$$= \sqrt{8^2 + 6^2}$$

$$= \sqrt{64 + 36}$$

$$= \sqrt{100}$$

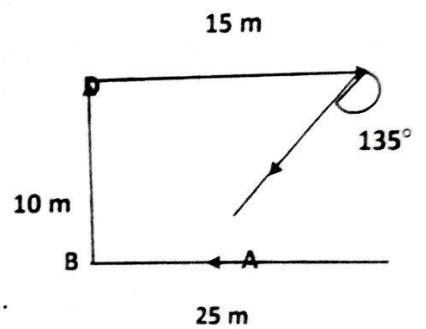
$$= 10\ Km$$

29. P started from his house towards west. After walking a distance of 25 m. He turned to the right and walked 10 m. he then again turned to the right and walked 15 m. after this he is to turn at $135°$ and to cover 30 m. In which direction should he go?

Ans: South-West

Explanation:

C

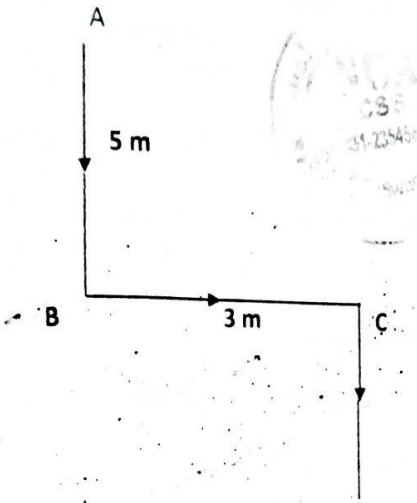


Hence he should go in the South-west direction

30. X started to walk towards straight towards south. After walking 5m he turned to the left and walked 3 m. After this he turned to right and walked 5 m now to which direction X is facing?

Ans: south

Explanation:

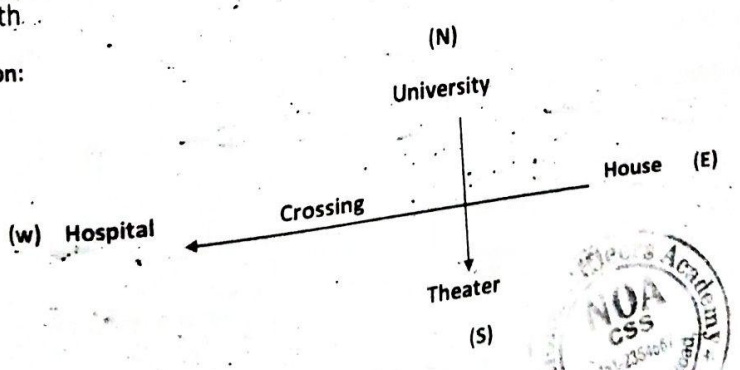


Hence X will face in the end towards South.

31. Hermant in order to go to university started from his house in the east and came to a crossing. The road to the left ends in a theatre, straight ahead is the hospital. In which direction is the university?

Ans: North

Explanation:



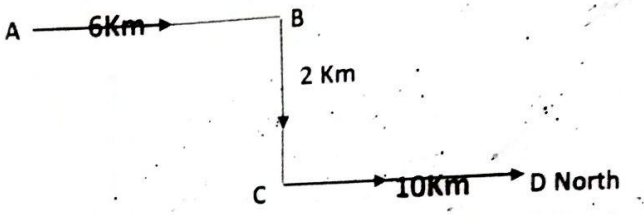
Therefore, University is in North.

Prepared by Azhar Afzal Mir

32. After walking 6 km, I turned to the right and then walked 2 Km. after then I turned to the left and walked 10 km. in the end, I was moving towards the north. From which direction did I start my journey?

Ans: south

Explanation:

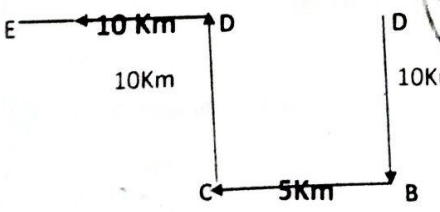


The journey was started from the South.

33. Ravi left home and cycled 10 km towards south, then turned right and cycled 5 km and then again turned right and cycled 10 km. After this he turned left and cycled 10 km. however many kilometers will he have to cycle to reach his home straight?

Ans: 15 km

Explanation:



$$\textbf{Required distance} = AE$$

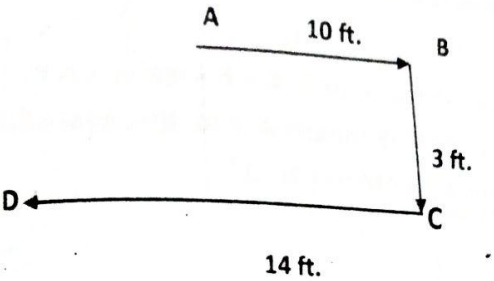
$$= 5 + 10$$

$$= 15\ km$$

34. Reena walked from A to B in the east 10 feet. Then she turned to the right and walked 3 feet. Again she turned to the right and walked 14 feet. How far is she from A?

Ans: 5 feet

Explanation:



$$Required\ distance = AD$$

$$= \sqrt{3^2 + (14 - 10)^2}$$

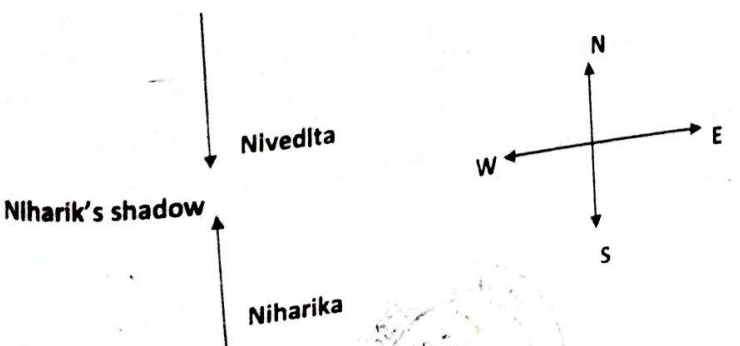
$$= \sqrt{9 + 16}$$

$$= 5\ ft.$$

35. One morning after sunrise Nivedita and Niharika were talking to each other face to face at Dalphin crossing. If Niharika's shadow was exactly to the right of Nivedita, which direction was facing?

Ans: North

Explanation:



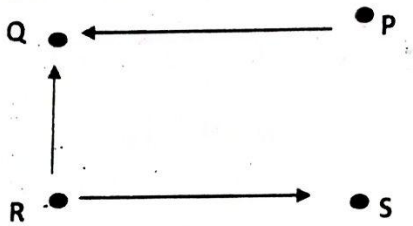
In the morning sun rises in The East. Hence then any shadow falls in the West. Since Niharrika's shadow was exactly to the right of Nivedita. Hence Niharika is facing towards north.

36. If A x B means A is to the south of B; A + B means A is to the north of B; A % B means A is to the east of B; A -B means A is to the west of B; then in P % Q + R -S,S is in which direction with respect to Q?

Ans: south-East

Explanation:

According to P%Q+R-S



37. One morning after sunrise, Suresh was standing facing a pole. The shadow of the pole falls exactly to his right. To which direction was he facing?

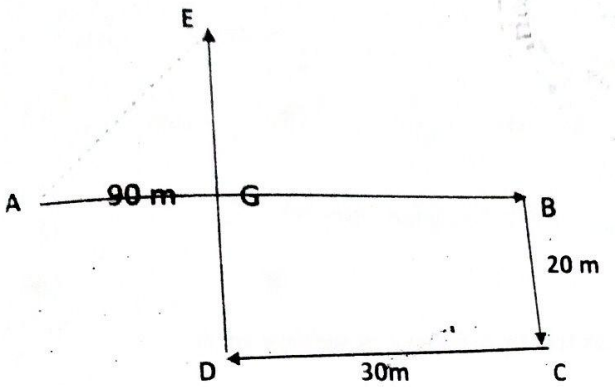
Ans: south

Explanation: Sun rises in the East in the morning. Since the shadow of Suresh falls to his right. So he is facing south.

38. A child went 90 m in the East to look for his father, then he turned right and went 20 m. after this he turned right and after going 30 m he reached to his uncle's house .His father was not there. From there he went 100 m to his north and met his father. How far did he meet his father from the starting point?

Ans: 100 m

Explanation:



$$\textbf{Required distance} = AE$$

$$= \sqrt{(AG)^2 + (EG)^2}$$

$$= \sqrt{(90-30)^2 + (100-20)^2}$$

$$= \sqrt{60^2 + 80^2}$$

$$= \sqrt{3600 + 6400}$$

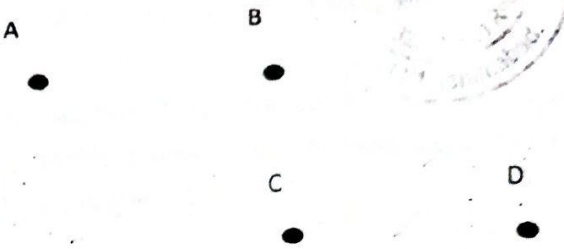
$$= \sqrt{10000}$$

$$= 100\ m$$

39. Four friends A, B, C and D live in a same locality. The house of B is in the east of A's house but in the north of C's house. The house of C is in the west of D's house. D's house is in which direction of A's house?

Ans: south-East

Explanation:

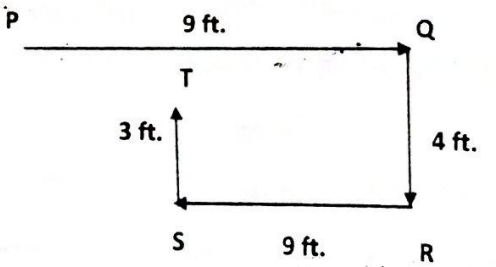


Therefore, D's house is in the South-East direction of A.

40. Umesh directly went from P, to Q which is 9 feet distant. Then he turns to the right and walked 4 feet. After this he turned to the right and walked a distance which is equal from P to Q. finally he turned to the right and walked 3 feet. How far is he now from P?

Ans: 1 feet

Explanation:



$$\text{Required distance} = PT$$

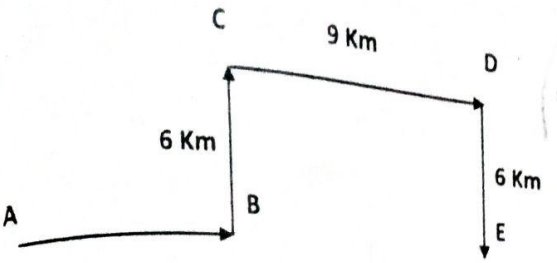
$$= 4 - 3$$

$$= 1 \text{ ft.}$$

41. Shyam walks 5 km towards East and then turns left and walk 6 km. Again he turns right and walks 9 km. Finally he turns to his right and walk 6 km. How far is he from the starting point?

Ans: 14 km

Explanation:



$$Required\ distance = AE$$

$$= 5 + 9$$

$$= 14\ Km$$

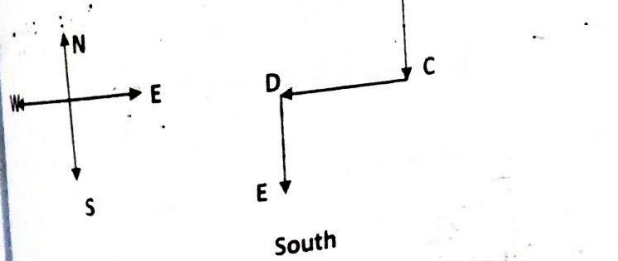
42. Amit started walking positioning his back towards the sun. After some time, he turned left, then turned right and towards the left again. In which direction is he going now?

3

Ans: North or South.

Explanation:

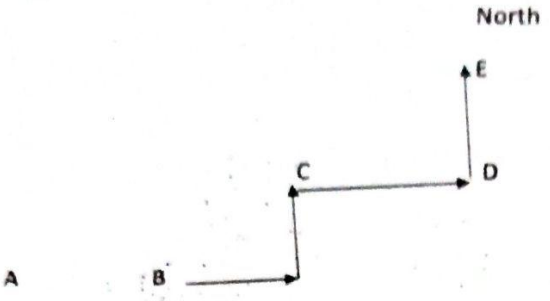
In morning



South

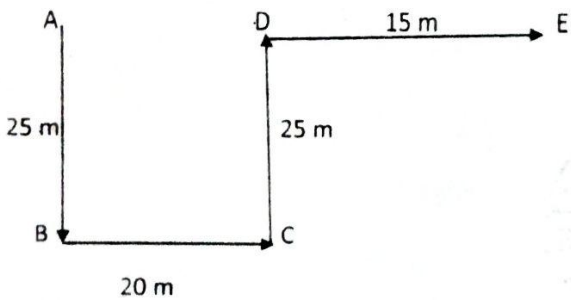
gain he
ow far

In evening



43. Rohit walked 25 m towards south. Then he turned to his left and walked 20 m. He then turned to his left and walked 25 m. He again turned to his right and walked 15 m. at what distance is he from the starting point and in which direction?

Ans: 35 m East



$$Required\ distance = AE$$

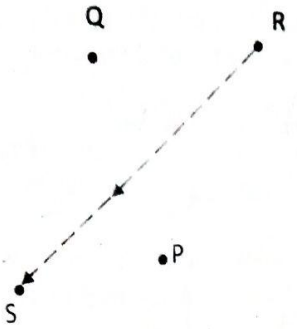
$$= 20 + 15$$

$$35\ m\ towards\ east.$$

44. Village Q is to the North of the village P .The village R is in the east of village Q.The village S is to the left of the village P. in which direction is the village S with respect to village R?

Ans: south-west

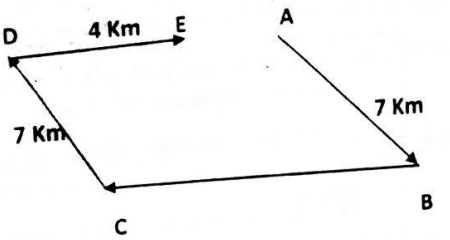
Explanation:



45. **Radha moves towards south-east a distance of 7 km, then she move towards west and travels a distance of 14 km. From here she moves toward north-west a distance of 7 km and finally she moves distance of 4 km towards east. How far is she now form the starting point?**

Ans: 10 km

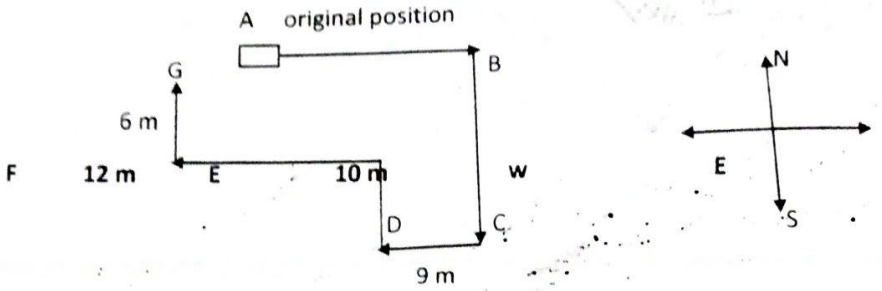
Explanation:



$$\text{Required distance} = AE$$
$$= 14 - 4$$
$$= 10 \text{ Km}$$

46. **Sundar runs 20 m towards East and turns to right and runs 10 m. Then he turns to the right and runs 9 m. Again he turns to right and runs 5 m. After this he turns to left and runs 12 m and finally he turns to right and 6m. Now to which direction is Sundar facing?**

Ans: North



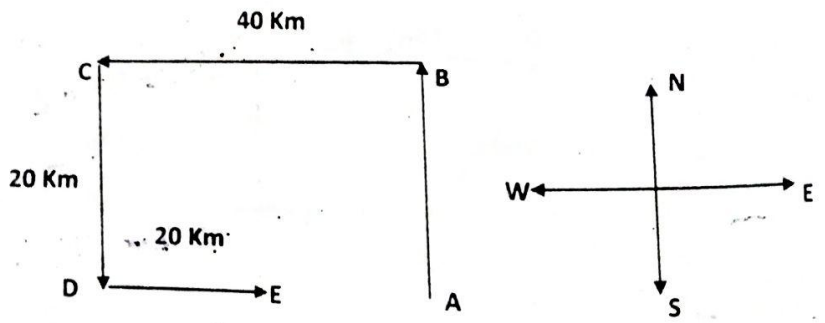
Therefore, it's clear that the Sundar will face towards North.

47. **Sachin walks 20 km towards North. He turns left and walk 40 km. he again turns left and walks 20 km. finally he moves 20 km after turning to the left.**

How far is he from his starting position?

Ans: 20 km

Explanation:



Required distance = 40 – 20 = 20Km.